
Optimal Taxation

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Lecture 7

Frank Plumpton Ramsey (February 22, 1903 – January 19, 1930) was a British mathematician, philosopher and economist.

Ramsey's immortal contribution to economic theory was the elegant concept of Ramsey pricing. This is applicable in situations where a (regulated) monopolist wants to maximize consumer surplus whilst at the same time ensuring that its costs are adequately covered. This is achieved by setting the price such that the markup over marginal cost is inversely proportional to the price elasticity of demand for that good.

Today's Menu

Optimal Commodity Taxation → I will derive the Ramsey rule using two different methods.

From the second method, I will derive the well-known inverse elasticity rule of taxation.

Discussion on optimal income taxation when equity is added to the equation.

Optimal Taxation

This chapter is really an extension of the material in the previous chapter on efficiency.

This is the question we will attempt to address in this chapter:

If taxes have an excess burden how should taxes be designed so as to minimize the excess burden (and maximize govt revenue) ?

Optimal Commodity Taxation

The chapter in the text begins with a result that I already derived : that a tax on all commodities *including leisure* would be a tax on the value of a person's time endowment, and would therefore be lump–sum.

In this case, the inability to impose a lump sum tax is irrelevant.

The government can effectively take away a lump sum amount through *equal taxes on all commodities (including leisure)*.

No excess burden.

Optimal Commodity Taxation

Obviously we need to proceed under the more realistic assumption that such a lump-sum tax is infeasible since taxing leisure is virtually impossible to implement.

As a result, some excess burden is inevitable. Key question is how to select rates on "X" and "Y" to minimize excess burden subject to achieving the government revenue constraint.

Optimal Commodity Taxation: The Ramsey Rule (1)

To answer this, consider the following, somewhat artificial problem : given the total excess burden, let's try and raise as much revenue as possible.

That is, suppose that we raise the tax on food by one percent, and then lower the tax on clothing by whatever amount makes the total excess burden constant.

This policy, by construction, leaves the excess burden constant : it just substitutes more excess burden in the food market for less in the clothing market.

The Ramsey Rule (2)

It's worth doing if the new policy — higher taxes on food, lower taxes on clothing — leads to more tax revenue (TR). Then we've got more revenue for the same amount of waste. By construction,

$$\Delta EB^F = \Delta EB^C$$

The move is a good one if

$$\Delta TR^F < \Delta TR^C$$

The Ramsey Rule (3)

That is, dividing both sides by the equivalent changes in excess burden, if at the margin

$$\frac{\Delta TR^F}{\Delta EB^F} < \frac{\Delta TR^C}{\Delta EB^C}$$

This inequality suggests that it's a good idea to raise the tax on food and lower the tax on clothing, since, at the margin, the excess burden per dollar of tax raised is higher in the clothing market.

If the inequality goes the other way, then food taxation brings greater waste per dollar raised, so it's better, at the margin, to lower the tax on food and raise the tax on clothing.

The Ramsey Rule (4)

We should adjust the tax mix until the excess burden per dollar of revenue raised, at the margin, is equal across commodities.

So that says that an optimal commodity tax system should be one for which $\Delta TR/\Delta EB$ is the same for every commodity.

Otherwise we could raise the revenue yield for a given excess burden (or lower the excess burden for a given revenue requirement) by adjusting taxes, namely by raising them on goods for which $\Delta TR/\Delta EB$ is low.

Deriving the Ramsey Rule (ΔEB)

One of my approximate formulae for the area of the excess burden triangle is

$$EB \approx (0.5)t(\Delta Q) \approx -(0.5)t^2 \frac{\partial Q}{\partial t} \quad (1)$$

If I took the slope of the compensated demand curve **as constant**, then the derivative of this with respect to t is

$$\Delta EB \approx -t \frac{\partial Q}{\partial t} \approx \Delta Q \quad (2)$$

Notice I can actually get the first part of this without approximation by using the “area equals integral” formula

$$EB = \int_p^{p+t} Q(P)dP - tQ(p+t)$$

and differentiating this with respect to t . (NOT DONE HERE).

Deriving the Ramsey Rule (ΔTR)

The tax revenue is

$$TR = tQ$$

so that

$$\frac{\partial TR}{\partial t} = Q + t \frac{\partial Q}{\partial t} \approx \Delta TR \quad (3)$$

Ramsey Rule: Expressing $(\Delta TR/\Delta EB)$

$$\frac{\Delta TR}{\Delta EB} = \frac{Q - \Delta EB}{\Delta EB} = \frac{Q}{\Delta EB} - 1 \quad (4)$$

So I get $\Delta TR^F / \Delta EB^F = \Delta TR^C / \Delta EB^C$ if

$$\frac{Q^F}{\Delta Q^F} = \frac{Q^C}{\Delta Q^C}$$

This is equation 17.9 of the text, the first version of the so-called Ramsey rule for taxation (after the English economist who first derived it).This rule is true EVEN IF goods are related (complements or substitutes.)

Interpreting Equation 17.9

If the commodity tax system minimizes the excess burden of raising a given amount of revenue, then it should reduce demand for all goods by the same **proportion**.

Notice that this rule does **not** say that the proportional tax *rate* on each good should be the same. It says that the proportional reduction in demand for each good, caused by the whole tax system, should be the same for each good.

Quite generally, a commodity tax system will minimize the total excess burden only if the quantity purchased of each good is reduced by the same proportion — approximately, and if people were compensated for the damage done by the tax.

Alternative way of looking at the Ramsey Rule (1).

We continue, for simplicity, to assume that goods are not related in consumption.

Let's write the optimization problem consisting in minimizing excess burden under the constraint of reaching a certain tax yield.

$$\min_{t_X, t_Y, \lambda} L = \left[\frac{1}{2} \eta_X P_X X t_X^2 \right] + \left[\frac{1}{2} \eta_Y P_Y Y t_Y^2 \right] + \lambda [M - P_X X t_X - P_Y Y t_Y]$$

Let's solve this problem.

Alternative way of looking at the Ramsey Rule (2)

Solving leads to a relationship between tax rates and elasticities:

$$t_X \eta_X^c = t_Y \eta_Y^c$$

Or rearranging we have the inverse elasticity rule:

$$\frac{t_X}{t_Y} = \frac{\eta_Y^c}{\eta_X^c}$$

Summing Up our Results (1)

The “inverse elasticity” formula gives a very precise and specific prescription for commodity taxation. It says that the commodity tax system should not be uniform — unless all goods had the same compensated own–price elasticity of demand.

It does not tell me immediately how high all these rates should be, just the relationship among them. But the revenue needs of the government will dictate how high the absolute level of these taxes should be.

But there is one, potentially very big, catch here. This formula only works if demand for each of the taxed goods is independent of the prices of the other taxed goods.

The Corlett-Hague rule

Corlett and Hague looked at a more restricted problem : suppose that there already is a tax in place, and that we cannot alter that tax.

What does the presence of that first tax imply for the tax rates which we can control. Chapter 17 tells me the answer: other things equal, we should raise the taxes on substitutes to the taxed good, and lower the taxes on complements.

As an important example, consider the fact that we levy a lot of taxes on labour income in Canada. What does the presence of those taxes imply for what we should do with commodity taxes?

The Corlett-Hague rule

A tax on labour income is really a subsidy on leisure which also yield an excess burden. So to reduce that excess burden, complements to leisure should be taxed, and substitutes taxed less.

So taxation of goods we consume with our leisure (movies, video games, cottages) will help reduce the excess burden of the taxation of labour income.

Taxation of complement to labour (work–related software, lunch buckets) should be discouraged, since those taxes would increase (at least in theory) the already–existing distortion, towards too much leisure.

Equity Issue...

It is certainly worth mentioning, again, that excess burden is entirely an efficiency problem.

One of Ramsey's rules, the inverse elasticity rule, suggests that food be taxed at a very high rate, since it does appear to have a very low compensated own-price elasticity of demand.

But of course food also has a very low income elasticity of demand. A tax on food does fall disproportionately on low-income people.

The analysis of Ramsey's basic model completely ignores distributional consequences, which is a very serious defect.

After all, if we truly did not care about redistribution, then we could levy head taxes, which are entirely feasible, which are lump-sum and which therefore have no excess burden.

Optimal Income Taxation

Optimal income taxation is quite a different problem than optimal commodity taxation. In optimal commodity taxation the issue was which commodities to tax, and at what rates. By definition, optimal income taxation is a problem in taxing income, so we already know what we are taxing.

The question then becomes what should the income tax schedule look like. In other words, how progressive should the income tax be? This problem brings out quite starkly the potential conflict between equity and efficiency.

Trade-off between Equity and Efficiency (1)

Suppose that we need to raise a given amount of money from the income tax.

Suppose further that I propose to make the burden of the income tax on the poor less, by raising the basic exemption. Suppose that I raise the level of income which is not subject to tax.

This change will make the income tax burden much less at low income. If I raise the basic exemption from \$10,000 to \$12,000, (and then tax all income above the exempt amount at 17 percent), then I'd reduce the tax paid by a family earning \$15,000 from \$850 to \$510, a pretty large percentage reduction.

Trade-off between Equity and Efficiency (2)

Raising the exemption will lower the tax yield. To make up the revenue shortfall, what can I do?

Raise the tax collected from higher income people by raising the marginal rate they pay. However, my formula for the excess burden said that it increases with the square of the tax rate.

Raising the marginal rate will definitely increase the excess burden of the income tax, and the numbers from the end of chapter 16 suggested that excess burden was already pretty high.

So there is a conflict here, and a tough choice : higher marginal rates mean a more progressive system, but also mean a greater excess burden.

The question in optimal income tax theory is how to weigh these costs and benefits, and what factors would lead to the costs of more progressivity being higher or lower.

Optimal Income Taxation & Equity

To account for distributional (equity) issues, the Ramsey rule has been modified.

Degree of departure from original rule depends on:

1. How much society cares about equity
2. Extent to which consumption patterns of rich and poor differ

Edgeworth's Contribution (1)

The British economist Edgeworth recognized this result about a century ago. he asked : what would be the best income tax system, if income were truly exogenous?

In particular, he wanted to know how the progressivity of the income tax would depend on people's (or the government's) attitude toward inequality.

In general, we might think that a social welfare function had some aversion to inequality : transferring a dollar from a rich person to a poor person would raise social welfare, other things equal.

But there can be a wide range of views on how much inequality matters. Is a dollar more to a poor person the same as \$1.20 to a rich person? \$2? \$5?

Edgeworth's Contribution (2)

We might expect, the higher is our aversion to inequality, that is the more we think transfers to the poor are more valuable than transfers to the rich, the more progressive we would want the income tax to be.

Edgeworth showed that conclusion is wrong. As long as you think income is worth more, socially, at the margin to the poor than to the rich, then the optimal income tax should involve a marginal rate of — wait for it — 100 percent.

It doesn't matter whether you weight transfers to the poor as a little more valuable than transfers to the rich, or a lot. Raising the marginal rate (and raising the exemption so as to keep the revenue yield constant) will transfer income from the rich to the poor.

Edgeworth's Contribution (3)

So if you do not think the ideal income tax system is one which confiscates everything and redistributes it equally, then probably one of two things are true :

1. Either you don't care at all about poverty or income inequality
2. Or you think that income taxes affect economic incentives, and that these incentives are important.

The analysis becomes more difficult if we drop the (not very realistic) assumption that income is exogenous.

Modern Studies: Endogenizing Labour Income

Now the average income has to be treated as a function of the tax system.

Assume a tax schedule characterize by

$$revenue = -\alpha + t \times Income$$

If the government needs to raise \$5000, then it must pick t and α so that

$$t[a(t, \alpha) - \alpha] = 5000$$

where $a(t, \alpha)$ is the average income in the country, which depends on the tax rate and on the level of the exemption (α).

Higher values of t mean more progressive tax but larger excess burdens.

Optimal income taxation comes down to find the right combination of α and t .

Stern Contribution (1974)

Solving this optimal flat tax problem, Stern showed that how progressive the income tax should be does depend on 3 factors:

1. Our attitude towards inequality (when income is endogenous).
2. On the substitutability of leisure for other goods
3. On the government's revenue requirement.

Stern's finds that a value of $t=20\%$ would maximize social welfare....far from Edgeworth 100%.

General Conclusion from Stern's model

Earlier, we mentioned that the excess burden of the income tax was proportional to the compensated elasticity of labour supply.

The bigger is that elasticity, the lower should be t .

The bigger is that elasticity, the higher the excess burden of the income tax, and the less progressive is the optimal income tax.

More substitutability between leisure and goods raises the marginal cost (in terms of higher excess burden) of a more progressive income tax.

The bigger is our aversion to inequality, the higher is the marginal benefit of more progressive income tax, and the higher is the optimal income tax rate.

Very Recent Contributions

There is no reason to restrict attention to income tax schedules with constant marginal tax rates. Economists have looked at the optimal overall shape of the income tax schedule, without restricting that shape to be linear.

That is, if we can vary the marginal rate by income class, what should be the optimal schedule?

It turns out that often it appears that it is not optimal to deviate much from a flat tax — although that conclusion is rather sensitive to the assumptions made.

Recent Contribution: Gruber and Saez (2000).

In a recent paper, Gruber and Saez (2000) showed that a tax system with different marginal rate by income class is optimal (and yet progressive), if people in higher-income tax brackets face lower marginal tax rates than people in lower brackets.

(Intuitive) Explanation of this Paradox

Lowering the marginal tax rate on high income people induce them to supply more Labour (L).

Increasing L increase tax revenue which can be used to lower the tax burden of low-income earners.

Although marginal tax rates fall with income, the average tax rates rise with income, so this optimal tax system is still progressive.

It is important to remember, that a lot of assumptions and value judgements lie behind all these results. These considerations are really at the core of the debate on optimal tax systems.
