TECHNICAL APPENDICES

A. Proof of Lemma 1

Proof. Assume Condition (1) is true.

1. **Proof of $\alpha_0^{0,t} < 0$.**

   Recall that $\alpha_0^{0\,t} = v_0^{0,t} + V_0^{0,t}$. We know,

   $$v_0^{0,t} = (w+K-1)(1+\beta) - [w + \beta E_t[\overline{p}_1^{t+1}(w+K-1-D_1^{1,t+1}) + (1-\overline{p}_1^{t+1})(w-s)]]$$

   where $E_t[.]$ denotes the expectation function based on information in period t, $\overline{p}_1^{t+1}$ is the probability that an older woman will find a match in the period $(t+1)$ and $D_i^{j,t+1}$ is the payment associated with a marriage of a woman of age i and man of age j in period $(t+1)$, $(i, j = 0, 1)$.

   Remark 1. *Notice that in the above expression I do not break $\overline{p}_1^{t+1}$ into the probabilities of matching with different types of men $\left(p_1^{0,t+1}\right.$ and $\left.p_1^{1,t+1}\right)$ in period $(t+1)$. This is not necessary since when agents are matched with more than one type in equilibrium, they must be indifferent between them (Shapley and Shubik (1972), Roth and Sotomayor (1990)).*

   Simplifying, we get

   $$v_0^{0,t} = (K-1) + \beta(K+s-1) - \beta E_t[\overline{p}_1^{t+1}(K+s-1-D_1^{1,t+1})]$$

   Suppose $\overline{p}_1^{t+1} < 1$. Then $D_1^{1,t+1} = (K+s-1)$ $(= v_1^{1,t+1}$, because old women in $(t+1)$ must be indifferent between marrying and not marrying). Hence, we can derive

   $$v_0^{0,t} = (K-1) + \beta(K+s-1) \qquad (A.1)$$

   Suppose $\overline{p}_1^{t+1} = 1$. Then we get

   $$v_0^{0,t} = (K-1) + \beta E_t D_1^{1,t+1} \qquad (A.2)$$

   Now consider $V_0^{0,t}$:

   $$V_0^{0,t} = (w+K-1)(1+\beta) - [w + \beta E_t[\overline{q}_1^{t+1}(w+K-1+D_1^{1,t+1}) + (1-\overline{q}_1^{t+1})(w-s)]]$$

where $\overline{q}_1^{t+1}$ is the probability that an older man will find a match in the period $(t+1)$.

Simplifying, we obtain

$$V_0^0 = (K-1) + \beta(K+s-1) - \beta E_t[\overline{q}_1^{t+1}(K+s-1+D_1^{1,t+1})]$$

Suppose $\overline{q}_1^{t+1} < 1$. Then $D_1^{1,t+1} = -(K+s-1)$ ($= -V_1^{1,t+1}$ because old men in $(t+1)$ must be indifferent between marrying and not marrying). Then,

$$V_0^0 = (K-1) + \beta(K+s-1) \tag{A.3}$$

Suppose $\overline{q}_1^{t+1} = 1$. Then,

$$V_0^{0,t} = K - 1 - \beta E_t D_1^{1,t+1} \tag{A.4}$$

Now, let $\overline{p}_1^{t+1} < 1$; then $\overline{q}_1^{t+1} = 1$ (else, the marriage market does not clear). Hence, $ED_1^{1,t+1} = K+s-1$. Then (using $(A.1)$ and $(A.4)$),

$$\begin{aligned} \alpha_0^{0,t} &= 2(K-1) \\ &\Rightarrow \alpha_0^{0,t} < 0 \text{ when } K < 1 \end{aligned}$$

Alternatively, let $\overline{p}_1^{t+1} = 1$ and $\overline{q}_1^{t+1} < 1$. Then $ED_1^{1,t+1} = -(K+s-1)$. Then (using $(A.2)$ and $(A.3)$),

$$\begin{aligned} \alpha_0^{0,t} &= 2(K-1) \\ &\Rightarrow \alpha_0^{0,t} < 0 \text{ when } K < 1 \end{aligned}$$

Finally, suppose $\overline{p}_1^{t+1} = 1$ and $\overline{q}_1^{t+1} = 1$. Suppose the expected dowry for $(1,1)-$marriages is $ED_1^{1,t+1}$. Then (using $(A.2)$ and $(A.4)$),

$$\begin{aligned} \alpha_0^{0,t} &= 2(K-1) \\ &\Rightarrow \alpha_0^{0,t} < 0 \text{ when } K < 1 \end{aligned}$$

Hence, when $K < 1$ (Condition (1)), $\alpha_0^{0,t} < 0$.

Remark 2. *Notice from the proof above, that in general, the probability of matching in any period* ($\overline{p}_1^t$ *or* $\overline{q}_1^t$) *may be replaced by* 1. *This is because when* $\overline{p}_1^t < 1$ ($\overline{q}_1^t < 1$), *the equilibrium dowry is such that the old woman (old man) is reduced to her (his) outside option* $(w - s)$ *in period* t. *That is, in a period–*t *equilibrium,*

$$\overline{p}_1^t(w + K - 1 - D_1^{1,t+1}) + (1 - \overline{p}_1^t)(w - s) \equiv w + K - 1 - D_1^{1,t}$$

and

$$\overline{q}_1^t(w + K - 1 - D_1^{1,t}) + (1 - \overline{q}_1^t)(w - s) \equiv w + K - 1 - D_1^{1,t+1}$$

2. **Proof of** $\alpha_1^{1,t} > \alpha_0^{1,t} > 0$**.**

   Recall $\alpha_1^{1,t} = v_1^{1,t} + V_1^{1,t} \Rightarrow$

$$\alpha_1^{1,t} = 2(K + s - 1) \tag{A.5}$$

Also, we can express $\alpha_0^{1,t} = V_0^{1,t} + v_0^{1,t}$ as

$$\alpha_0^{1,t} = 2K + s + \beta + \beta E_t D_1^{1,t+1} \tag{A.6}$$

$(A.6)$ implies that the maximum value $\alpha_0^{1,t,\max}$ corresponds to the maximum (feasible) value $ED_1^{1,t+1,\max} = v_1^{1,t+1} = K + s - 1$. Hence,

$$\alpha_0^{1,t,\max} = 2K + s(1 + \beta) + K\beta \tag{A.7}$$

Condition (1) imposes $s > \frac{\beta+2}{1-\beta}$. Since $\frac{\beta+2}{1-\beta} > \frac{K\beta+2}{1-\beta}$ (because $K < 1$), we also have $s > \frac{K\beta+2}{1-\beta}$. This implies the following:

$$\begin{aligned} s &> \frac{K\beta + 2}{1 - \beta} \\ &\Rightarrow s(1 - \beta) > K\beta + 2 \\ &\Rightarrow (2K + s - 2) + s(1 - \beta) > (2K + s - 2) + K\beta + 2 \\ &\Rightarrow 2(K + s - 1) > 2K + K\beta + s(1 + \beta) \end{aligned}$$

The above directly implies (using $(A.5)$ and $(A.7)$),

$$\alpha_1^{1,t} > \alpha_0^{1,t,\max}$$

$$\Rightarrow \alpha_1^{1,t} > \alpha_0^{1,t}$$

Note also, that

$$\begin{aligned} s &> \frac{K\beta + 2}{1-\beta} > \frac{K\beta - 2(K+\beta)}{1-\beta} \\ &\Rightarrow s(1-\beta) > K\beta - 2(K+\beta) \\ &\Rightarrow 2K + s + \beta - \beta(K + s - 1) > 0 \end{aligned}$$

Since $ED_1^{1,t+1,\min} = -V_1^{1,t+1} = -(K+s-1)$, the above directly implies

$$2K + s + \beta + \beta ED_1^{1,t+1,\min} > 0$$

But, using $(A.6)$, this means:

$$\alpha_0^{1,t,\min} > 0$$

$$\Rightarrow \alpha_0^{1,t} > 0$$

3. **Proof of** $\alpha_1^{1,t} > \alpha_1^{0,t}$.

Recall $\alpha_1^{0,t} = v_1^{0,t} + V_1^{0,t}$, simplified to yield

$$\alpha_1^{0,t} = 2K + s - \beta - 4 - \beta E_t D_1^{1,t+1}$$

Hence,

$$\begin{aligned} \alpha_1^{0,t,\max} &= 2K + s - \beta - 4 - \beta E_t D_1^{1,t+1,\min} \\ \Rightarrow \alpha_1^{0,t,\max} &= 2K - 2\beta - 4 + K\beta + s(1+\beta) \qquad (A.8) \end{aligned}$$

since $ED_1^{1,t+1,\min} = -V_1^{1,t+1} = -(K+s-1)$.

Note that

$$\frac{K\beta - 2\beta - 2}{1-\beta} < 0$$

since $K < 1, 0 < \beta < 1$ (from Condition (1) and model assumptions).

Therefore,

$$\begin{aligned} s &> 0 > \frac{K\beta - 2\beta - 2}{1-\beta} \\ &\Rightarrow s(1-\beta) > K\beta - 2\beta - 2 \\ &\Rightarrow 2K + 2s - 2 > 2K - 2\beta - 4 + K\beta + s + s\beta \end{aligned}$$

The above implies (directly from $(A.5)$ and $(A.8)$):

$$\alpha_1^{1,t} > \alpha_1^{0,t,\max}$$
$$\Rightarrow \alpha_1^{1,t} > \alpha_1^{0,t}$$

■

B. Stable Population Equilibrium

B.1. Derivation of mapping ϕ

$$\begin{aligned} u_{old,00}^t &= 0 \\ u_{old,11}^t &= u_{00}^{t-1} \end{aligned} \tag{B.1}$$

where $u_{old,ij}^t$ is the measure of already-married couples (i,j) at the beginning of period t and u_{ij}^t is the measure of (i,j) couples at the end of period t.

Define,

$$\begin{aligned} u_{old,01}^t &= u_{10}^{t-1} \\ & (the\ measure\ of\ old\ males\ who\ when\ young,\ married\ old\ females) \\ u_{old,10}^t &= u_{01}^{t-1} \\ & (the\ measure\ of\ old\ females\ who\ when\ young,\ married\ old\ males) \end{aligned} \tag{B.2}$$

Then

$$\begin{aligned} u_{00}^t &= u_{old,00}^t + u_{new,00}^t \\ u_{11}^t &= u_{old,11}^t + u_{new,11}^t \\ u_{01}^t &= u_{new,01}^t \\ u_{10}^t &= u_{new,10}^t \end{aligned} \tag{B.3}$$

where $u_{new,ij}^t$ is the measure of new matches (i,j) – specified by the matching rule μ_{ij} – made in period t from among the eligible agents $(f_0^t, f_1^t, m_0^t, m_1^t)$. That is,

$$u_{new,ij}^t = \mu_{ij}(f_0^t, f_1^t, m_0^t, m_1^t) \tag{B.4}$$

In each period single agents $(f_0^t, m_0^t, f_1^t, m_1^t)$ are determined as follows:

$$\begin{aligned} f_0^t &= F_0^t - u_{old,00}^t & (B.5) \\ m_0^t &= M_0^t - u_{old,00}^t \\ f_1^t &= F_1^t - u_{old,10}^t - u_{old,11}^t \\ m_1^t &= M_1^t - u_{old,01}^t - u_{old,11}^t \end{aligned}$$

$(B.1) - (B.5)$ define a mapping ϕ^u:

$$u_{old}^t = \phi^u(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1}) \qquad (B.A)$$

The measure of newborns in each period is given by (recall that old unions do not produce children):

$$\begin{aligned} F_0^t &= \sum_i \sum_j b_{ij} u_{new,ij}^{t-1} = \sum_i \sum_j b_{ij} \mu_{ij}(f_0^{t-1}, f_1^{t-1}, m_0^{t-1}, m_1^{t-1}) \\ M_0^t &= \sigma \sum_i \sum_j b_{ij} u_{new,ij}^{t-1} = \sigma \sum_i \sum_j b_{ij} \mu_{ij}(f_0^{t-1}, f_1^{t-1}, m_0^{t-1}, m_1^{t-1}) \end{aligned}$$

Using $(B.5)$, the above can be written as

$$\begin{aligned} F_0^t &= \sum_i \sum_j b_{ij} \mu_{ij}(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1}) \\ M_0^t &= \sigma \sum_i \sum_j b_{ij} \mu_{ij}(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1}) \\ Also,\ F_1^t &= F_0^{t-1} \\ M_1^t &= M_0^{t-1} \end{aligned}$$

The above defines a mapping ϕ^1:

$$i.e.\ (F_0^t, F_1^t, M_0^t, M_1^t) = \phi^1(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1}) \qquad (B.B)$$

$(B.A)$ and $(B.B)$ define the mapping ϕ:

$$(F_0^t, F_1^t, M_0^t, M_1^t, u_{old}^t) = \phi(F_0^{t-1}, F_1^{t-1}, M_0^{t-1}, M_1^{t-1}, u_{old}^{t-1})$$

B.2. Proof of Proposition 1

Proof. Suppose that the total population is in a stable population equilibrium, growing at the rate $(1+\widehat{r})$. The result follows from Definition 6 and equation $(B.5)$ in Appendix B.1. ■

C. Proof of Lemma 2 & Corollary 1

C.1. Proof of Lemma 2

Proof. (In the proof below, steady-state values of variables are denoted without a time superscript, indicating that these values are the same in every period. ED_i^j denotes the expected dowry in an $(i,j)-$marriage in every period.)

Assume Condition (1) holds. Then young men do not marry young women and old agents are matched before young agents (Lemma 1 and (7)). Assume also that sex ratios are exogenous.

Assume that young men are willing to marry old women at the payments offered. Keeping this in mind, there can be 7 possible demographic configurations that can be sustained in every period of a steady state equilibrium. These are:

1. $f_1^t > m_1^t + m_0^t$ $(m_1^t = 0)$
2. $f_1^t = m_1^t + m_0^t$ $(m_1^t = 0)$
3. $m_1^t < f_1^t < m_1^t + m_0^t$
4. $m_1^t = f_1^t < m_1^t + m_0^t$
5. $f_1^t < f_1^t + f_0^t = m_1^t < m_1^t + m_0^t$ $(f_1^t = 0)$
6. $f_1^t < f_1^t + f_0^t < m_1^t < m_1^t + m_0^t$ $(f_1^t = 0)$
7. $f_1^t < m_1^t < m_1^t + m_0^t < f_1^t + f_0^t$

Focus on configurations $(1)-(4)$ in which $f_1^t \geq m_1^t$. Below, I show the following:

i Configurations (1) and (2) are inconsistent with a non-trivial steady state equilibrium, since $m_1^t = 0$ and $\alpha_i^0 < 0$ (i.e. there are no old men and young men are not willing to marry at the equilibrium payments, leaving women without partners). This leaves configurations $(3)-(7)$ as the only possible steady state configurations (Corollary 1).

ii When configurations (3) or (4) are are true in a non-trivial steady state equilibrium, $\alpha_1^0 < 0$. This proves Lemma 2.

Proof of part (i):

Consider configuration (1). The configuration reduces to $f_1^t > m_0^t$ since in each period there are more old women than men implying that all men must be married young; hence $m_1^t = 0$. Since some old women do not find a match the (steady state) equilibrium dowry is

$$D_1^0 = v_1^0 = (K + s - 2) \tag{$C.1$}$$

Note that the above dowry reduces old women to indifference between marrying versus not marrying.

Young men's surplus from marriage, V_1^0, can be derived to be

$$V_1^0 = K - 2 - \beta - \beta E D_1^1 \tag{$C.2$}$$

In steady state, the expected dowry $ED_1^1 = D_1^1 = (K + s - 1)$ (making old women indifferent to the age of the groom); hence $(C.2)$ implies

$$V_1^0 = K - 2 - K\beta - \beta s$$

Using $(C.1)$ and $(C.2)$, we obtain the (equilibrium) value of a marriage between young men and old women ($\alpha_1^0 = v_0^1 + V_0^1$) to be:

$$\alpha_1^0 = 2K - 4 - K\beta + s(1 - \beta) \tag{$C.3$}$$

Condition (1) imposes $s < \frac{\beta+2+2(1-K)(1-\beta)}{1-\beta} = \frac{4+2K\beta-2K-\beta}{1-\beta}$. Since $\frac{4+K\beta-2K}{1-\beta} > \frac{4+K\beta-2K-\beta(1-K)}{1-\beta} = \frac{4+2K\beta-2K-\beta}{1-\beta}$ (because $K < 1$), we know $s < \frac{4+K\beta-2K}{1-\beta}$. This implies that $\alpha_1^0 < 0$ (see $(C.3)$). In other words, young men are not willing to marry old women in the steady state equilibrium. But configuration (1) implies $m_1^t = 0$. Hence, there are no old men in the marriage market and the young men are not willing to marry either type of woman. This implies that the population must reduce to zero (trivial equilibrium).

Hence, configuration (1) cannot be sustained in a non-trivial steady state equilibrium.

Consider configuration (2). The configuration reduces to $f_1^t = m_0^t$ since in each period there are enough old women for all men implying that all men must be married young; hence $m_1^t = 0$. There must be multiple equilibria in payments, which are assumed to follow a uniform distribution. That is,

$$D_1^0 \sim U[-V_1^0, v_1^0]$$

Hence, the expected dowry is given by

$$ED_1^0 = \frac{-V_1^0 + v_1^0}{2} \tag{$C.4$}$$

Simplifying (using $(C.1), (C.2), (C.4)$), we get

$$ED_1^0 = \frac{s + \beta + \beta ED_1^1}{2} \tag{$C.4'$}$$

Note that in any period old women are indifferent between marrying young men and old men (if there were any) if

$$\begin{aligned} v_1^1 - ED_1^1 &= v_1^0 - ED_1^0 \\ \Rightarrow ED_1^1 &= ED_1^0 + 1 \end{aligned} \tag{$C.5$}$$

Together, $(C.4')$ and $(C.5)$ imply

$$ED_1^0 = \frac{s + 2\beta}{2 - \beta} \tag{$C.6$}$$

Using $(C.1) - (C.2)$ and the equilibrium dowries $(C.5) - (C.6)$, we can obtain the magnitude of $\alpha_1^0 = v_1^0 + V_1^0$:

$$\alpha_1^0 = \frac{2[2K - K\beta - 4 + s(1 - \beta)]}{2 - \beta} \tag{$C.7$}$$

Condition (1) imposes $s < \frac{\beta+2+2(1-K)(1-\beta)}{1-\beta} = \frac{4+2K\beta-2K-\beta}{1-\beta}$. Since $\frac{4+K\beta-2K}{1-\beta} > \frac{4+K\beta-2K-\beta(1-K)}{1-\beta} = \frac{4+2K\beta-2K-\beta}{1-\beta}$ (because $K < 1$), we know $s < \frac{4+K\beta-2K}{1-\beta}$; hence $\alpha_1^0 < 0$. In other words, young men are not willing to marry old women in equilibrium. Also, there are no old men since $m_1^t = 0$. Hence, configuration (2) cannot be sustained in a non-trivial steady state equilibrium.

Proof of part (ii)**:**

Consider configuration (3): $m_1^t < f_1^t < m_1^t + m_0^t$. Assume $\alpha_1^0 \geq 0$. Then, in equilibrium, all old women find a match, some with young and some with old men. Some young men find a match with old women, and some do not marry at all (since they are not willing to marry young women). Therefore, young men must be indifferent between marrying old women or postponing marriage ($D_1^0 = -V_1^0$). This means that

$$D_1^0 = -[K - 2 - \beta - \beta D_1^1] \tag{$C.8$}$$

Old women are indifferent between marrying young and old men if:

$$\begin{aligned} v_1^1 - D_1^1 &= v_1^0 - D_1^0 \\ &\Rightarrow D_1^1 = 1 + D_1^0 \end{aligned} \tag{C.9}$$

From $(C.8)$ and $(C.9)$ we get:

$$\begin{aligned} D_1^0 &= \frac{2(1+\beta)-K}{1-\beta} \\ D_1^1 &= \frac{3+\beta-K}{1-\beta} \end{aligned} \tag{C.10}$$

Now consider the value of $\alpha_1^0 = v_1^0 + V_1^0$ at the equilibrium dowries $(C.10)$:

$$\alpha_1^0 = \frac{2K - K\beta + s(1-\beta) - 4}{1-\beta}$$

Condition (1) imposes $s < \frac{\beta+2+2(1-K)(1-\beta)}{1-\beta} = \frac{4+2K\beta-2K-\beta}{1-\beta}$. Since $\frac{4+K\beta-2K}{1-\beta} > \frac{4+K\beta-2K-\beta(1-K)}{1-\beta} = \frac{4+2K\beta-2K-\beta}{1-\beta}$ (because $K < 1$), we know $s < \frac{4+K\beta-2K}{1-\beta}$; hence $\alpha_1^0 < 0$. In other words, young men are not willing to marry old women in equilibrium. This is a contradiction, since the equilibrium dowries were derived assuming $\alpha_1^0 \geq 0$.

Now consider the equilibrium payments when $\alpha_1^0 < 0$.

Then old women must be indifferent between marrying and being single for life. Hence,

$$D_1^1 = K + s - 1$$

Consider the magnitude of $\alpha_1^0 = V_1^0 + v_1^0$ at the equilibrium dowry above:

$$\alpha_1^0 = 2K - K\beta + s(1-\beta) - 4$$

Here too, $\alpha_1^0 < 0$ (from Condition (1)). This is consistent with the initial assumption $\alpha_1^0 < 0$.

Hence, in a steady state equilibrium of configuration (3), young men do not marry.

Suppose that configuration (4) holds in steady state equilibrium: $m_1^t = f_1^t < m_1^t + m_0^t$. There can be multiple equilibria in payments with the limits being determined by what makes old men and old women indifferent to their outside option from the marriage.

Assume that $\alpha_1^0 \geq 0$.

Then, the outside option of old women is marriage to young men. Also, the outside option of old men is marriage to young women.

Using $(C.2)$, we obtain the dowry that old women must pay in order to marry a young man, $D_1^0 (= -V_1^0)$ to be

$$D_1^0 = -[K - 2 - \beta - \beta E D_1^1]$$

The highest dowry that old women will pay old men in this period must be the one that makes them indifferent between marrying old and young men. Hence,

$$\begin{aligned} v_1^0 - D_1^0 &= v_1^1 - D_1^{1,\max} \\ &\Rightarrow D_1^{1,\max} = 1 - [K - 2 - \beta - \beta E D_1^1] \end{aligned}$$

The highest bride price that men will pay in this period depends on their outside option from marrying young women. To marry young women, old men must be willing to accept the payment, D_0^1, where

$$\begin{aligned} D_0^1 &= v_0^1 \\ &\Rightarrow D_0^1 = K + \beta + \beta E D_1^1 \end{aligned}$$

The highest bride price (or lowest dowry) that old men will pay old women will be the one that makes them indifferent between marrying old and young women. Hence,

$$\begin{aligned} V_0^1 + D_0^1 &= V_1^1 + D_1^{1,\min} \\ &\Rightarrow D_1^{1,\min} = 1 + K + \beta + \beta E D_1^1 \end{aligned}$$

Dowries are assumed to follow a uniform distribution over the range $[D_1^{1,\min}, D_1^{1,\max}]$. Hence, the expected dowry is given by

$$\begin{aligned} ED_1^1 &= \frac{D_1^{1,\min} + D_1^{1.\max}}{2} \\ &\Rightarrow ED_1^1 = \frac{2+\beta}{1-\beta} \end{aligned} \qquad (C.11)$$

Now consider the magnitude of α_1^0 in the equilibrium (using $(C.11)$):

$$\alpha_1^0 = \frac{2K - 2K\beta - 4 + \beta + s(1-\beta)}{1-\beta} \qquad (C.12)$$

Condition (1) imposes $s < \frac{\beta+2+2(1-K)(1-\beta)}{1-\beta} = \frac{4+2K\beta-2K-\beta}{1-\beta}$. Hence, $\alpha_1^0 < 0$, and young men are not willing to marry old women at the equilibrium dowry. This is a contradiction since the equilibrium dowry was derived assuming $\alpha_1^0 \geq 0$.

Now assume $\alpha_1^0 < 0$. Then, the maximum dowry that old women will pay old men will be given by:

$$\widehat{D}_1^{1.\max} = v_1^1 = K + s - 1$$

This implies, in turn, that

$$\begin{aligned} ED_1^1 &= \frac{D_1^{1,\min} + \widehat{D}_1^{1.\max}}{2} \qquad (C.13) \\ &\Rightarrow \; ED_1^1 = \frac{2K + s + \beta}{2 - \beta} \end{aligned}$$

Now check the magnitude of α_1^0 at the ED_1^1 derived in $(C.13)$:

$$\alpha_1^0 = \frac{2[2K - 2K\beta - 4 + \beta + s(1-\beta)]}{2-\beta}$$

Condition (1) imposes $s < \frac{\beta+2+2(1-K)(1-\beta)}{1-\beta} = \frac{4+2K\beta-2K-\beta}{1-\beta} \Rightarrow \alpha_1^0 < 0$. This is consistent with the assumption of $\alpha_1^0 < 0$ under which the equilibrium dowries were derived.

Hence, in a steady state equilibrium of configuration (4), young men do not marry. ■

C.2. Proof of Corollary 1

Proof. The proof of part (i) presented above demonstrates that configurations $(1)-(2)$ are not compatible with a non-trivial steady state equilibrium when Condition (1) is true. This allows only configurations $(3)-(7)$ to be sustained as potential steady state configurations. Moreover, in configurations $(3)-(4)$, young men do not marry old women since $\alpha_1^0 < 0$. Under these conditions, it is easy to see that configurations $(3)-(7)$ above are identical to configurations $(a)-(e)$ in equation $(**)$ (Corollary 1). ■

D. Proof of Proposition 2

D.1. Derivation of E_f and E_m

Let $\widetilde{E}_f^{t+1}$denote the *total* utility that a woman who attains marriageable age in period $(t+1)$ can generate over her lifetime. Then,

$$\begin{aligned}\widetilde{E}_f^{t+1} &= \overline{p}_0^{t+1}[(w+K)(1+\beta)-D_0^{1,t+1}]+(1-\overline{p}_0^{t+1})\overline{p}_1^{t+2}[w+\beta\{w+K-1-D_1^{1,t+2}\} \\ &\quad +\{1-\overline{p}_0^{t+1}-(1-\overline{p}_0^{t+1})\overline{p}_1^{t+2}\}[w+\beta(w-s)]\end{aligned}$$

where $\overline{p}_i^{t+k}$ is the probability that a woman of age i finds a partner in period $(t+k)$ and $D_i^{j,t+k}$ refers to the payment in a marriage between bride i and groom j in period $(t+k)$.

Remark 3. *Notice that in the above expression I do not break $\overline{p}_i^{t+k}$ into the probabilities of matching with different types of men in equilibrium $\left(p_i^{0,t+k} \text{ and } p_i^{1,t+k}\right)$. This is not necessary since when agents are matched with more than one type in equilibrium, they must be indifferent between them (Shapley and Shubik (1972), Roth and Sotomayor (1990)). When matched with only one type in equilibrium, $\widetilde{E}_g^{t+1}$ must contain the probability of matching with this type and the returns from this marriage. Since old agents are matched first (Lemma 1), it is sufficient to use the returns from these matches (i.e. with old partners) in the expression for $\widetilde{E}_g^{t+1}$.*

If a woman cannot find a partner in her lifetime, she gets $w+\beta(w-s)$ (she receives w in the first period and $(w-s)$ in the second period, due to the cost of being single in old age). Hence the total marriage *surplus* that a daughter conceived in period t is expected to generate over her lifetime (denoted E_f^{t+1}) is

$$\begin{aligned}E_f^{t+1} &= E_t[\widetilde{E}_f^{t+1}]-[w+\beta(w-s)] \qquad (D.1)\\ &= E_t[\overline{p}_0^{t+1}[K(1+\beta)+\beta s-D_0^{1,t+1}]+\beta(1-\overline{p}_0^{t+1})\overline{p}_1^{t+2}[K+s-1-D_1^{1,t+2}]]\end{aligned}$$

By a similar derivation,

$$E_m^{t+1}=E_t[\overline{q}_0^{t+1}[(K-2)(1+\beta)+\beta s+D_1^{0,t+1}]+\beta(1-\overline{q}_0^{t+1})\overline{q}_1^{t+2}[K+s-1+D_1^{1,t+2}] \qquad (D.2)$$

where E_m^{t+1} is the total surplus that a man conceived in period t is expected to generate over this (marriageable) lifetime and $\overline{q}_j^{t+k}$ is the probability that a man of age j finds a partner in period $(t+k)$.

D.2. Proof of Proposition 2

Proof. (For ease of exposition, time superscripts are dropped in this proof.) The optimization problem in (12) – (13) (see text) may be solved by the Lagrangean method. The Lagrangean function is

$$L = c + E_f b_f + E_m b_m - (b_f - b_m)^2 + \lambda[\rho_i - b_f - b_m] + \pi_f b_f + \pi_m b_m$$

The first order conditions are:

$$L_f = \frac{\partial L}{\partial b_f} = E_f - 2(b_f - b_m) - \lambda + \pi_f = 0 \qquad (D.3)$$

$$L_m = \frac{\partial L}{\partial b_m} = E_m + 2(b_f - b_m) - \lambda + \pi_m = 0 \qquad (D.4)$$

$$\lambda L_\lambda = \lambda(\rho_i - b_f - b_m) = 0; \lambda \geq 0, (\rho_i - b_f - b_m) \geq 0 \qquad (D.5)$$

$$\pi_f L_{\pi_f} = \pi_f b_f = 0; \pi_f \geq 0,\ b_f \geq 0 \qquad (D.6)$$

$$\pi_m L_{\pi_m} = \pi_m b_m = 0; \pi_m \geq 0,\ b_m \geq 0 \qquad (D.7)$$

Suppose $\lambda > 0$ and $\pi_f = \pi_m = 0$ (i.e. $b_f > 0,\ b_m > 0$ *and* $b_f + b_m = \rho_i$).

The interior solution is obtained as

$$\begin{aligned} b^*_{fi} &= \frac{4\rho_i + (E_f - E_m)}{8} \\ b^*_{mi} &= \frac{4\rho_i - (E_f - E_m)}{8} \end{aligned} \qquad (D.8)$$

Suppose parameters are such that $b^*_{fi} > 0,\ b^*_{mi} > 0$. Let us check that the assumption of $\lambda > 0$ is justified. The marginal utilities of having a boy or a girl at (b^*_f, b^*_m) are given by:

$$\frac{\partial U^{marr}}{\partial b_f} = \frac{\partial U^{marr}}{\partial b_m} = \frac{E_f + E_m}{2} \geq 0$$

It is clearly optimal to conceive as many children as total fertility allows as long as the marginal utility of the additional child is strictly positive. Note that the marginal utility of the additional child will be zero only if $E_f = E_m = 0$.

This can occur when both agents have a zero probability of finding a match in their lifetime and may be true only when the population is reduced to zero, viz. a 'trivial' stable population. E_f (or E_m) may also be zero if there are so many women (men) in the population that they are forced to pay out their entire surplus as marriage payment in each period. However, when this is the case, men (women) find matches with ease and get paid positive dowries (bride price), hence E_m $(E_f) > 0$. Thus in all meaningful and non-trivial equilibria, the marginal utility of offspring is positive for interior solutions $(D.8)$ and couples will produce as many children as the fertility of the mother will allow.

Consider, without loss of generality, a corner solution in which $b_{mi} = 0,\ b_{fi} > 0$. Is the constraint on total fertility $(D.5)$ binding?

A binding constraint $(D.5)$ and $b_{mi} = 0$ implies $b_{fi} = \rho_i$. At this b_{fi}, $\frac{\partial U^{marr}}{\partial b_f} = E_f - 2\rho_i$. Note also that for $b_{mi} = 0$, we must have $4\rho_i - (E_f - E_m) < 0$ (see $(D.8)$). But this implies that $\rho_i < \frac{E_f - E_m}{4} < \frac{E_f}{2} - (\frac{E_f + E_m}{4}) < \frac{E_f}{2}$. Hence, $\frac{\partial U^{marr}}{\partial b_f} = E_f - 2\rho_i > 0$ and the constraint $(D.5)$ does indeed bind for corner solutions in which offspring of only one sex are desired in equilibrium.

The total fertility constraint is not binding in the case where $b_{fi} = b_{mi} = 0$. However, the only steady state equilibrium that is compatible with $b_{fi} = b_{mi} = 0$ is the trivial equilibrium.

Hence, in all non-trivial population equilibria, couples have as many children as total fertility allows. This implies that younger women have more children than older women, since fertility depends only on the age of the mother and younger mothers are more fertile.

Let us look at the conditions under which constraints $(D.6)$ and $(D.7)$ are binding. Since $\rho_1 < \rho_0$, when $|E_f - E_m| > 4\rho_0$, all agents choose $b_{mi} = 0$ if $E_f > E_m$ or $b_{fi} = 0$ if $E_f < E_m$. This cannot be compatible with a steady state equilibrium for the following reason. Suppose, without loss of generality, that $E_f > E_m$, so that $|E_f - E_m| > 4\rho_0$ implies that all agents choose to have baby girls. This will wipe out boys from the population reducing E_f to zero (because the girls cannot find matches) and violating the condition that $E_f > E_m$. When $|E_f - E_m| < 4\rho_1$, both constraints $(D.6)$ and $(D.7)$ will fail to bind and an interior solution such as in $(D.8)$ is obtained. When $4\rho_1 < |E_f - E_m| < 4\rho_0$ the high fertility couples (with young women) will have offspring of both sexes but the low fertility agents have offspring of one sex only (viz. the one with the higher E_g).

Using $(D.8)$, taking simple derivatives will show that the optimal male-to-female sex ratio σ_i (where i is the mother's age) varies inversely with women's excess marriage market returns $(E_f - E_m)$. Also, σ_i can be shown to increase (decrease) with declines in total fertility ρ_i if $E_f - E_m < 0$ $(E_f - E_m > 0)$.

This yields the result stated in Proposition 2. ■

E. Example 3: Derivation

Assume the following parameter values: $\rho_0 = 3,\ \rho_1 = 2,\ K = 0.2,\ s = 4,\ \beta = 0.25$. It is straightforward to show that Condition (1) is true at these values, hence Lemma 1 and Lemma 2 are true. The following claims demonstrate the existence of a non-trivial steady state general equilibrium at the assumed values (time superscripts are dropped, representing steady state values).

Claim 1. *In a stable population equilibrium,* $\overline{q}_0 = 0,\ \overline{q}_1 = 1,\ \overline{p}_0 = 0.787,\ \overline{p}_1 = 1$. *The stable population grows at the rate* $(1 + \widehat{r}) = 1.237$, *so* $\widehat{r} = 0.237$.

Proof. This is computationally demonstrated by graphing the evolution of the population given the parameter values. Table (a) at the end of the appendices shows graphs of the evolution of the population beginning from some initial vector of marriage market participants. I have presented graphs of $\frac{F_0^{t+1}}{F_0^t}$, $\frac{M_0^{t+1}}{M_0^t}$, $\frac{f_1^{t+1}}{f_1^t}$ and $\frac{m_1^{t+1}}{m_1^t}$ (since the assumed mortality rates imply $F_1^t = F_0^{t-1}$, $M_1^t = M_0^{t-1}$ and the fact that all newborns are single ensure $F_0^t = f_0^t$, $M_0^t = m_0^t$). Note that the equilibrium values of $\overline{q}_j$ and $\overline{p}_i$ $(i, j = 0, 1)$ imply that $f_1^t < m_1^t < f_1^t + f_0^t$ in the steady state equilibrium. ■

Claim 2. *The equilibrium marriage payments are* $D_0^1 = \frac{K+2\beta}{1-\beta} = 0.93$, $D_1^1 = \frac{K+1+\beta}{1-\beta} = 1.93$. *Hence the equilibrium marriage payments are dowries.*

Proof. Let $\overline{q}_0 = 0,\ \overline{q}_1 = 1,\ \overline{p}_0 = 0.787,\ \overline{p}_1 = 1$. Within-group competition for spouses among young women must reduce their marriage surplus to zero. So,

$$\begin{aligned} v_0^1 - D_0^1 &= 0 \qquad (E.1) \\ &\Rightarrow \quad D_0^1 = K(1+\beta) - \beta(K - 1 - ED_1^1) \\ &\Rightarrow \quad D_0^1 = \frac{K + 2\beta}{1 - \beta} = 0.93 \end{aligned}$$

(since in a steady state general equilibrium, $D_i^j = ED_i^j$ for all i, j and $D_0^1 = 1 + D_1^1$ so that old men are indifferent to the age of their spouse). Hence,

$$D_1^1 = D_0^1 + 1 = \frac{K + 1 + \beta}{1 - \beta} = 1.93 \qquad (E.2)$$

The payments are positive, hence dowries. ■

Claim 3. *The optimal maternal-age-specific birth rates and sex ratios are:* $b_{f0} = 1.38,\ b_{m0} = 1.62,\ \sigma_0 = 1.17,\ b_{f1} = 0.88,\ b_{m1} = 1.12,\ \sigma_1 = 1.27$

Proof. Using equations $(D.1)$ and $(D.2)$ (in Appendix D.1), and substituting parameter values, we obtain $E_f = 0.5694675$ and $E_m = 1.5325$. Since $|E_f - E_m| < 4\rho_1$, there will be an interior solution to the problem of sex ratio choice (see proof of Proposition 2 in Appendix D.2). The solution follows from equation $(D.8)$ of Appendix D.2. ■

F. Proof of Proposition 3

$(**)$ (in Corollary 1) lists the five possible demographic configurations that may be obtained in a non-trivial steady state equilibrium. Recall that $K > 0, s > 0$ and $0 < \beta < 1$ and assume throughout that Condition (1) holds; hence Lemma 1 and Lemma 2 are true. (In the proof below, time superscripts are dropped, representing steady state values.)

Proof. Case (a): Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $f_1^t > m_1^t > 0$ for all t.

Since $(*)$ implies that old women are matched first, none of the young women are matched in equilibrium, i.e. $\overline{p}_0 = 0$. Also, since young men choose to postpone marriage (Lemma 2) and all young agents survive to old age, we have $f_1^t = F_1^t = F_0^{t-1}$ and $m_1^t = M_1^t = M_0^{t-1}$. Hence $f_1^t > m_1^t$ implies $F_0^{t-1} > M_0^{t-1}$, so

$$\sigma^{t-1} = \frac{M_0^{t-1}}{F_0^{t-1}} < 1$$

Since the population is in a stable equilibrium, growing at $\widehat{r}$, the overall sex ratio must be a constant σ in all periods. Hence,

$$\begin{aligned} \sigma &< 1 \qquad \mathit{for\ all\ t} && (F.1) \\ i.e.\ \sigma &< (1 - \overline{p}_0) + (1 + \widehat{r}) \end{aligned}$$

since $\overline{p}_0 = 0, (1 + r) \geq 0$.

Equilibrium marriage payments are

$$D_1^1 = K + s - 1 \qquad (F.1')$$

due to within-group competition among older women who pay their entire marital surplus as payment.

Note that Condition (1) imposes $s > \frac{2+\beta}{1-\beta}$ and $\frac{2+\beta}{1-\beta} > 2+\beta$ (since $0 < \beta < 1$). Hence, $s > 2+\beta \Rightarrow s > 2 \Rightarrow K+s-1 > 0$. The equilibrium payment in $(F.1')$ is therefore a dowry.

Case (b): Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $f_1^t = m_1^t < f_1^t + f_0^t$ for all t.

By an argument similar to the one above we can show that this implies

$$\begin{aligned} \sigma &= 1 \qquad\qquad (F.2)\\ i.e.,\ \sigma &< (1-\overline{p}_0) + (1+\widehat{r}) \end{aligned}$$

since $\overline{p}_0 = 0, (1+r) \geq 0$.

Since the numbers of old men and old women are perfectly matched, there will be multiple equilibria in marriage payments. The lower limit of payments D_1^1 will be the dowry that makes men indifferent to the age of women. This may be derived as

$$\begin{aligned} \underline{D}_1^1 &= 1 + D_0^1 = 1 + v_0^1 \\ or,\ \underline{D}_1^1 &= 1 + K + \beta + \beta E D_1^1 \end{aligned}$$

The upper limit is the dowry that reduces old women's surplus to zero. This is derived as

$$\overline{D}_1^1 = K + s - 1$$

Therefore,

$$D_1^1 \,\epsilon\, [1 + K + \beta + \beta E D_1^1,\ K + s - 1]$$

Assuming that marriage payments draw from a uniform distribution over the above range, we have $ED_1^1 = \frac{\overline{D}_1^1 + \underline{D}_1^1}{2}$, which reduces to

$$ED_1^1 = \frac{2K + s + \beta}{2 - \beta} \qquad (F.2')$$

The expected payment is a dowry because it is positive.

Case (c): Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $f_1^t + f_0^t = m_1^t$ for all t.

In such an equilibrium, $f_1^t = 0$ since all young women find a match in every period, i.e. $\overline{p}_0 = 1$. Also, since $f_0^t = F_0^t = (1+\widehat{r})F_0^{t-1}$ and $m_1^t = M_0^{t-1}$, this implies

$$\begin{aligned} (1+\widehat{r}) &= \sigma \qquad (F.3) \\ i.e.,\ \sigma &= (1-\overline{p}_0) + (1+\widehat{r}) \end{aligned}$$

since $\overline{p}_0 = 1$.

There will be multiple equilibria in marriage payments since the numbers of marriage market participants are exactly matched. The upper limit of D_0^1 is the dowry that reduces young women's marital surplus to zero. The lower limit is the bride price that reduces old men's surplus to zero. Deriving these in a manner similar to Case (b) and assuming that marriage payments draw from a uniform distribution over the permissible range, we obtain

$$D_0^1 \sim U[-(K+s),\ K(1+\beta) - \beta(K-2-ED_0^1)] \qquad (F.3')$$

$$\begin{aligned} ED_0^1 &= \frac{2\beta - s}{2-\beta} \\ ED_1^1 &= 1 + ED_0^1 = \frac{2+\beta-s}{2-\beta} \end{aligned}$$

Recall that Condition (1) implies $s > \frac{2+\beta}{1-\beta} > 2+\beta > 2\beta$ (since $\beta \in (0,1)$). Since $s > 2+\beta$ and $s > 2\beta$, the expected equilibrium payments in $(F.3')$ are negative; hence, bride price.

Case (d): Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $m_1^t > f_1^t + f_0^t$ for all t.

By an argument similar to the one above we can show that this implies

$$\begin{aligned} \sigma &> (1+\widehat{r}) \qquad (F.4) \\ i.e.,\ \sigma &> (1-\overline{p}_0) + (1+\widehat{r}) \end{aligned}$$

since $\overline{p}_0 = 1$.

Equilibrium marriage market payments are

$$D_0^1 = -(K+s) \qquad (F.4')$$

since old men engage in within group competition to secure a spouse. The equilibrium payment is negative, hence bride price.

Case (e): Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $f_1^t < m_1^t < f_1^t + f_0^t$ for all t.

Here only some young women find a match in every period, i.e. $\overline{p}_0 \in (0,1)$. Since $m_1^t = M_1^t = M_0^{t-1}$, $f_0^t = F_0^t = (1+\widehat{r})F_0^{t-1}$ and $f_1^t = (1-\overline{p}_0)F_0^{t-1}$, this implies

$$\sigma < (1-\overline{p}_0) + (1+\widehat{r}) \tag{$F.5$}$$

The equilibrium payments may be derived, as in $(E.1)$ and $(E.2)$ (in Appendix E), to yield

$$\begin{aligned} D_0^1 &= \frac{K+2\beta}{1-\beta} > 0 \qquad (F.5') \\ D_1^1 &= \frac{K+1+\beta}{1-\beta} > 0 \end{aligned}$$

Hence the marriage payments corresponding to (e) are dowries.

The following table summarizes the demographics and equilibrium payments in each of the cases $(a)-(e)$, when σ is exogenous.

Demographic Configuration	Expected Payment	$\overline{p}_0$	Inequality
$(a)\ f_1^t > m_1^t > 0$	*Dowry*	0	$\sigma < (1+\widehat{r}) + (1-\overline{p}_0)$
$(b)\ f_1^t = m_1^t < f_1^t + f_0^t$	*Dowry*	0	$\sigma < (1+\widehat{r}) + (1-\overline{p}_0)$
$(c)\ f_1^t < f_1^t + f_0^t = m_1^t$	*Bride* Pr*ice*	1	$\sigma = (1+\widehat{r}) + (1-\overline{p}_0)$
$(d)\ f_1^t < f_1^t + f_0^t < m_1^t$	*Bride* Pr*ice*	1	$\sigma > (1+\widehat{r}) + (1-\overline{p}_0)$
$(e)\ f_1^t < m_1^t < f_1^t + f_0^t$	*Dowry*	$\in\ (0,1)$	$\sigma < (1+\widehat{r}) + (1-\overline{p}_0)$

The expected equilibrium payment is dowry when $\sigma < (1+\widehat{r}) + (1-\overline{p}_0)$ and bride price when $\sigma \geq (1+\widehat{r}) + (1-\overline{p}_0)$. This proves Proposition 3. ■

G. Proof of Proposition 4

Proof. (Time superscripts are dropped in the proof below, indicating steady state values.)

Consider **Case (a)**: Using equations $(D.1)$ and $(D.2)$ (in Appendix D.1), we can derive

$$\begin{aligned} E_f &= 0,\ E_m = 2\beta(K+s-1) \qquad (G.1) \\ E_f - E_m &= -2\beta(K+s-1) \end{aligned}$$

at the equilibrium marriage payments (equation $(F.1')$).

Condition (1) imposes $s > \frac{2+\beta}{1-\beta}$ and $\frac{2+\beta}{1-\beta} > 2+\beta > 2$ (since $0 < \beta < 1$). Hence, $s > 2$ and $K > 0 \Rightarrow (K+s-1) > 0 \Rightarrow E_f - E_m < 0$.

Since we are interested only in non-trivial equilibria, let $|E_f - E_m| < 4\rho_0$ as outlined in Proposition 2.

It is easy to show that $b_{f_i} < b_{m_i}$ for each i, when $E_f - E_m < 0$ (Proposition 2). Hence, in aggregate $\sigma > 1$. This contradicts the condition that $\sigma < 1$ in Case (a) (see equation $(F.1)$). Hence there cannot be a steady state general equilibrium of the form (a).

Consider **Case (b)**: We can derive, as before, that at the equilibrium payments (see $(F.2')$),

$$E_f - E_m = -2\beta(\frac{2K + s + \beta}{2 - \beta}) < 0$$

As before, $E_f - E_m < 0$ implies that in a non-trivial equilibrium, $b_{f_i} < b_{m_i}$ for each i, hence $\sigma > 1$. This contradicts the fact that $\sigma = 1$ in Case (b) (see equation $(F.2)$). Hence there cannot be a steady state general equilibrium of the form (b).

Consider **Case (c)**: Since $f_1^t = 0$, the only marriage market participants are young women and old men. Hence, in equilibrium,

$$(F_0^{t+1}, M_0^{t+1}) = [((1 + \widehat{r})F_0^t, (1 + \widehat{r})M_0^t] = [b_{f0}F_0^t, b_{m0}F_0^t]$$

since F_0^t young women are matched in every period t. Hence, $1 + \widehat{r} = b_{f0}$. This implies (from equation $(F.3)$)

$$\begin{aligned} \sigma &= b_{f0} \qquad (G.2) \\ or,\ b_{m0} &= b_{f0}^2 \end{aligned}$$

Condition (1) implies $s > \frac{2+\beta}{1-\beta} > 2 + \beta > 2\beta$, since $0 < \beta < 1$. Hence, Condition (1) implies

$$s > 2\beta \qquad (G.3)$$

Now, using the equilibrium marriage payments ($(F.3')$), I derive

$$E_f - E_m = K - \frac{(2\beta - s)}{2 - \beta}(1 + \beta) \qquad (G.4)$$

$(G.3)$ and $(G.4)$ imply that

$$E_f - E_m > 0 \qquad (G.4')$$

In a non-trivial equilibrium, we have (recall that $f_1^t = 0$),

$$\begin{aligned} b_{f0} &= \frac{4\rho_0 + (E_f - E_m)}{8} \qquad (G.5) \\ b_{m0} &= \frac{4\rho_0 - (E_f - E_m)}{8} \end{aligned}$$

Denote $E_f - E_m = E$. Subsituting $(G.5)$ in $(G.2)$ yields

$$E^2 + (8 + 8\rho_0)E + (16\rho_0^2 - 32\rho_0) = 0 \qquad (G.6)$$

$(G.4')$ shows that $E > 0$, so the relevant root of $(G.6)$ for an equilibrium of the form (c) must be $E_1 = -(4 + 4\rho_0) + 4(1 + 4\rho_0)^{0.5} > 0$.

Note also that the following condition must be true in equilibrium[1]:

$$E_1 < Min[4\rho_0, 8 - 4\rho_0] \qquad (G.7)$$

Claim 4. *There do not exist parameter values* $K > 0$, $\beta \, \epsilon \, (0, 1)$ *and* $s > 0$ *that are consistent with Condition* (1) *and also with a steady state general equilibrium of the form* (c).

Proof. Suppose $E = E_1$ as must be true in an equilibrium. From $(G.4)$, we have:

$$K = E_1 + \frac{(2\beta - s)}{2 - \beta}(1 + \beta) \qquad (G.8)$$

In equilibrium, $(G.8)$ must be satisfied for some $K > 0$. Hence, in equilibrium,

$$E_1 + \frac{(2\beta - s)}{2 - \beta}(1 + \beta) > 0 \qquad (G.9)$$

A necessary condition for $(G.9)$ to hold is that it be true at the maximum value of E_1. It is easily shown that E_1 is maximized at $\rho_0^{\max} = 0.75$ and $E_1(\rho_0^{\max}) = 1$[2].

Hence, the necessary condition reduces to

$$2\beta^2 + (1 - s)\beta + 2 - s > 0 \qquad (G.10)$$

---

[1] $(E_1 < 4\rho_0)$ since we are interested in non-trivial equilibria and $(4\rho_0 + E_1 < 8)$, since $b_{f0} = \frac{4\rho_0 + E_1}{8} < 1$. This is because $E > 0$ implies $\sigma < 1$ (from $(G.5)$) and $b_{f0} = \sigma$ (from $(G.2)$).

[2] Note that $(G.7)$ is satisfied at $\rho_0^{\max}$.

The left hand side of $(G.10)$ is decreasing in s. Let $s = \frac{2+\beta}{1-\beta}$ (the lowest value of s permitted by Condition (1); this corresponds to the highest value of the left hand side).

Then, it is easy to show that for $(G.10)$ to be true at $s = \frac{2+\beta}{1-\beta}$, we must have

$$\frac{-2\beta^3 - 4\beta}{1-\beta} > 0$$

Clearly, the above inequality is not satisfied in the range $\beta \,\epsilon\, (0,1)$. ■

I have shown that even when ρ_0 is such that E attains the highest possible value consistent with a non-trivial equilibrium, there does not exist an s that is consistent with Condition (1) and $\beta \,\epsilon\, (0,1)$. This implies that there do not exist a set of parameter values consistent with model assumptions and Condition (1), that allow a steady state general equilibrium of the form (c).

Consider **Case (d)**: Using an argument similar to that employed for Case (c), we can show that in Case (d),

$$b_{m0}^2 > b_{f0} \qquad (G.11)$$

The equilibrium payments $(F.4')$ yield

$$E = E_f - E_m = 2K + \beta(K+s) + s > 0 \qquad (G.12)$$

As in the previous case, b_{f0} and b_{m0} are given by $(G.5)$ in a non-trivial equilibrium. Arguments similar to that employed above will show that in equilibrium,

$$E^2 + (8\rho_0 + 8)E + 16\rho_0^2 - 32\rho_0 < 0 \qquad (G.13)$$

Also, as before, the following condition must also be true in equilibrium:

$$0 < E < Min[4\rho_0, 8 - 4\rho_0] \qquad (G.14)$$

Consider the equation:

$$x^2 + (8\rho_0 + 8)x + 16\rho_0^2 - 32\rho_0 = 0 \qquad (see\ (F.6)) \qquad (G.15)$$

It is easy to show that $(G.13)$ is satisfied with $E > 0$ (as implied by $(G.12)$) when

$$E \,\epsilon\, (0, x_1) \qquad (G.16)$$

where x_1 is the positive root of $(G.15)$; $x_1 = -(4 + 4\rho_0) + 4(1 + 4\rho_0)^{0.5}$

Note that $x_1 > 0$ when $\rho_0 < 2$, hence this is a necessary condition for the existence of equilibrium.

Using $(G.12)$, $(G.16)$ reduces to the following condition:

$$0 < E = 2K + \beta(K + s) + s < x_1 \tag{$G.17$}$$

Claim 5. *There do not exist parameter values* $K > 0$, $\beta \epsilon (0, 1)$ *and* $s > 0$ *that are consistent with Condition* (1) *and also with a steady state general equilibrium of the form* (d).

Proof. Since E is increasing in s (see $(G.12)$) it is sufficient to show that there do not exist parameter values that satisfy (the right inequality of) $(G.17)$ at the minimum value of s that Condition (1) permits. viz. $s = \frac{2+\beta}{1-\beta}$.

It is easy to show that at $s = \frac{2+\beta}{1-\beta}$, the condition $E < x_1$ is equivalent to the condition:

$$\frac{2}{1-\beta} + \frac{\beta^2(1-K) + 2K + \beta(3-K)}{1-\beta} < x_1 \tag{$G.18$}$$

$(G.18)$ cannot be true because (a) the left hand side is greater than 2 when $1 > K > 0$ (Condition (1)) and $1 > \beta > 0$ and (b) the right hand side is 1 at its maximum. (It is easily shown that x_1 attains a maximum of 1 at $\rho_0 = 0.75$. Note that the necessary conditions of $\rho_0 < 2$ and $(G.14)$ are satisfied at this ρ_0)). Further, $(G.18)$ has been derived from $(G.17)$ using the lowest possible s permissible by Condition (1). For larger s, the left hand side of $(G.18)$) will be even higher, violating the requirement $E < x_1$ in $(G.17)$. ■

Hence, there do not exist a set of parameter values consistent with model assumptions and Condition (1), that allow a steady state general equilibrium of the form (d).

Hence, when Condition (1) is true, the only possible steady state general equilibrium is of the form (e).

Consider **Case (e):** Suppose that there is a stable population equilibrium (growing at $\widehat{r}$) in which $f_1^t < m_1^t < f_1^t + f_0^t$ for all t.

Example 3 (see Appendix E for a detailed derivation) illustrates a steady state general equilibrium of this form.

At the equilibrium payments,

$$E_f - E_m = -\frac{2\beta(K + \beta + 1)}{1-\beta} < 0$$

Therefore, in all non-trivial equilibria, $b_{fi} < b_{mi}$ (see proof of Proposition 3) so

$$\sigma > 1$$

Hence, an equilibrium of the form (e) is characterized by dowry (see ($F.5'$) and a male–to-female sex ratio at birth that is greater than 1.

An analysis of case (e) in Appendix F shows that $\sigma < (1 - \overline{p}_0) + (1 + \widehat{r})$, at this equilibrium. ■

H. Proof of Proposition 5 (and Lemma 3)

H.1. Proof of Lemma 3

Lemma 4. *Suppose that the post-marriage utility function of agents is given by* (14). *In all non-trivial steady state general equilibria, couples choose to have as many offspring as their total fertility allows, ensuring that young mothers have more offspring than old mothers. Maternal-age* (i)*-specific sex ratios* σ_i *(male/female) of offspring are determined as follows:*

when $|E_f - E_m| < 4\tau\rho_1$,

$$\sigma_i = \frac{4\tau\rho_i - (E_f - E_m)}{4\tau\rho_i + (E_f - E_m)} \epsilon\ (0, \infty)\ for\ i = 0, 1$$

when $4\tau\rho_1 < |E_f - E_m| < 4\tau\rho_0$,

$$\sigma_0 = \frac{4\tau\rho_0 - (E_f - E_m)}{4\tau\rho_0 + (E_f - E_m)} \epsilon\ (0, \infty)$$

$$\begin{aligned} \sigma_1 &= 0\ if\ E_m < E_f \\ \sigma_1 &= \infty\ if\ E_m > E_f \end{aligned}$$

Further, there is no non-trivial steady state equilibrium compatible with the condition $|E_f - E_m| > 4\tau\rho_0$.

Proof. *Follows from an argument identical to the one presented in the proof of Proposition 2 (see Appendix D.2). It is easy to see from the above derived expressions that* σ_i *increases (decreases) with declines in* τ *if* $E_f - E_m < 0$ ($E_f - E_m > 0$).

■

H.2. Proof of Proposition 5

Proof. Demographic configurations (a) and (b) (see $(**)$ in text) cannot be true in equilibrium for reasons identical to those presented in the proof of Proposition 4 (see Appendix G).

Consider an equilibrium of the form (**c**): $f_1^t + f_0^t = m_1^t$

Claim 6. *When $\tau < 1+\beta$, there do not exist parameter values $K > 0,\ \beta \epsilon (0,1)$ and $s > 0$ that are consistent with Condition* (1) *and also with a steady state general equilibrium of the form* (c).

Proof. Using an argument identical to that presented in Appendix G, we obtain that the following condition must be true in an equilibrium of the form (c) (see $(G.8)$):

$$K = E_1 + \frac{(2\beta - s)}{2-\beta}(1+\beta) \tag{$H.1$}$$

where $E_1 = -(4\tau + 4\tau\rho_0) + 4\tau(1+4\rho_0)^{0.5}$

Note that it is a necessary condition for equilibrium that $(H.1)$ be true for some $K > 0$ at the maximum possible value of E_1. It is easily shown that E_1 is maximized at $\rho_0^{\max} = 0.75$ and $E_1(\rho_0^{\max}) = \tau$. Hence $(H.1)$ reduces to

$$K = \tau + \frac{(2\beta - s)}{2-\beta}(1+\beta) \tag{$H.2$}$$

Now, $K > 0$ implies (using $(H.2)$)

$$2\beta^2 + (2-\tau-s)\beta - s + 2\tau > 0 \tag{$H.3$}$$

The left hand side of $(H.3)$ is decreasing in s. Let $s = \frac{2+\beta}{1-\beta}$ (the lowest value of s permissible by Condition (1)).

Then, some elementary algebra will demonstrate that a necessary condition for $(H.3)$ to be true at $s = \frac{2+\beta}{1-\beta}$, is

$$\tau > 1+\beta$$

Hence, the above necessary condition for the existence of a steady state of the form (c) is *not* satisfied when

$$\tau < 1+\beta \tag{$H.4$}$$

Alternatively, when $\tau < 1+\beta$, a steady state general equilibrium of the form (c) will not exist. ■

Consider a steady state general equilibrium of the form (**d**): $m_1^t > f_1^t + f_0^t$ for all t.

Claim 7. *When* $\tau < 2$, *there do not exist parameter values* $K > 0$, $\beta \in (0, 1)$ *and* $s > 0$ *that are consistent with Condition* (1) *and also with a steady state general equilibrium of the form* (d).

Proof. By an argument exactly similar to that presented in Appendix G, we obtain the following condition that must be true in an equilibrium of the form (d) (see $(G.17)$):

$$0 < E = 2K + \beta(K + s) + s < x_1 \qquad (H.5)$$

where $x_1 = -(4\tau + 4\tau\rho_0) + 4\tau(1 + 4\rho_0)^{0.5}$. Since E is increasing in s (see $(H.5)$) it is sufficient to show that at low values of τ, there do not exist parameter values that satisfy (the right inequality of) $(H.5)$.

Let $s = \frac{\beta+2}{1-\beta}$ (the lowest value of s permissible by Condition (1)). At $s = \frac{\beta+2}{1-\beta}$, $E < x_1$ (as in $(H.5)$) reduces to

$$\frac{2}{1-\beta} + \frac{K(2-\beta) + (1-K)\beta^2 + 3\beta}{1-\beta} < x_1 \qquad (H.6)$$

It is easily shown that x_1 attains a maximum of τ at $\rho_0 = 0.75$. Hence, the right hand side of $(H.6)$ is τ at its maximum, whereas the left hand side is greater than 2 for the relevant parameter range: $0 < K < 1$, $0 < \beta < 1$. Hence, there will be no equilibrium of the form (d) if $\tau < 2$. ■

Recall that a steady state general equilibrium of the form (c) cannot exist if $\tau < (1 + \beta)$. Since $(1 + \beta) < 2$ for $\beta \in (0, 1)$, a sufficient condition that ensures that equilibria of the form $(a) - (d)$ cannot be sustained in a steady state general equilibrium is that $\tau < (1 + \beta)$.

An equilibrium of the form (**e**) exists. Example: consider the following parameters: $K = 0.5$, $s = 5.2$, $\beta = 0.5$, $\rho_0 = 9$, $\rho_1 = 3$, $\tau = 0.75$. Notice that Condition (1) is satisfied and $\tau < 1 + \beta$. Then a steady state equilibrium of the form (e) is obtained that has the following characteristics:

$\overline{q}_0 = 0,\ \overline{q}_1 = 1,\ \overline{p}_0 = 0.47,\ \overline{p}_1 = 1,\ (1 + \widehat{r}) = 2.026$
$b_{f0} = 3.83,\ b_{m0} = 5.17,\ \sigma_0 = 1.35$
$b_{f1} = 0.83,\ b_{m1} = 2.17,\ \sigma_1 = 2.61$
$D_0^1 = \frac{K+2\beta}{1-\beta} = 3,\ D_1^1 = \frac{K+1+\beta}{1-\beta} = 4$

The proof is identical to that presented in Appendix E (Example 3), with birth rates being given by $b_{fi} = \frac{4\tau\rho_i + (E_f - E_m)}{8\tau}$, $b_{mi} = \frac{4\tau\rho_i - (E_f - E_m)}{8\tau}$. ■

Addendum to Appendix E: Table (a)

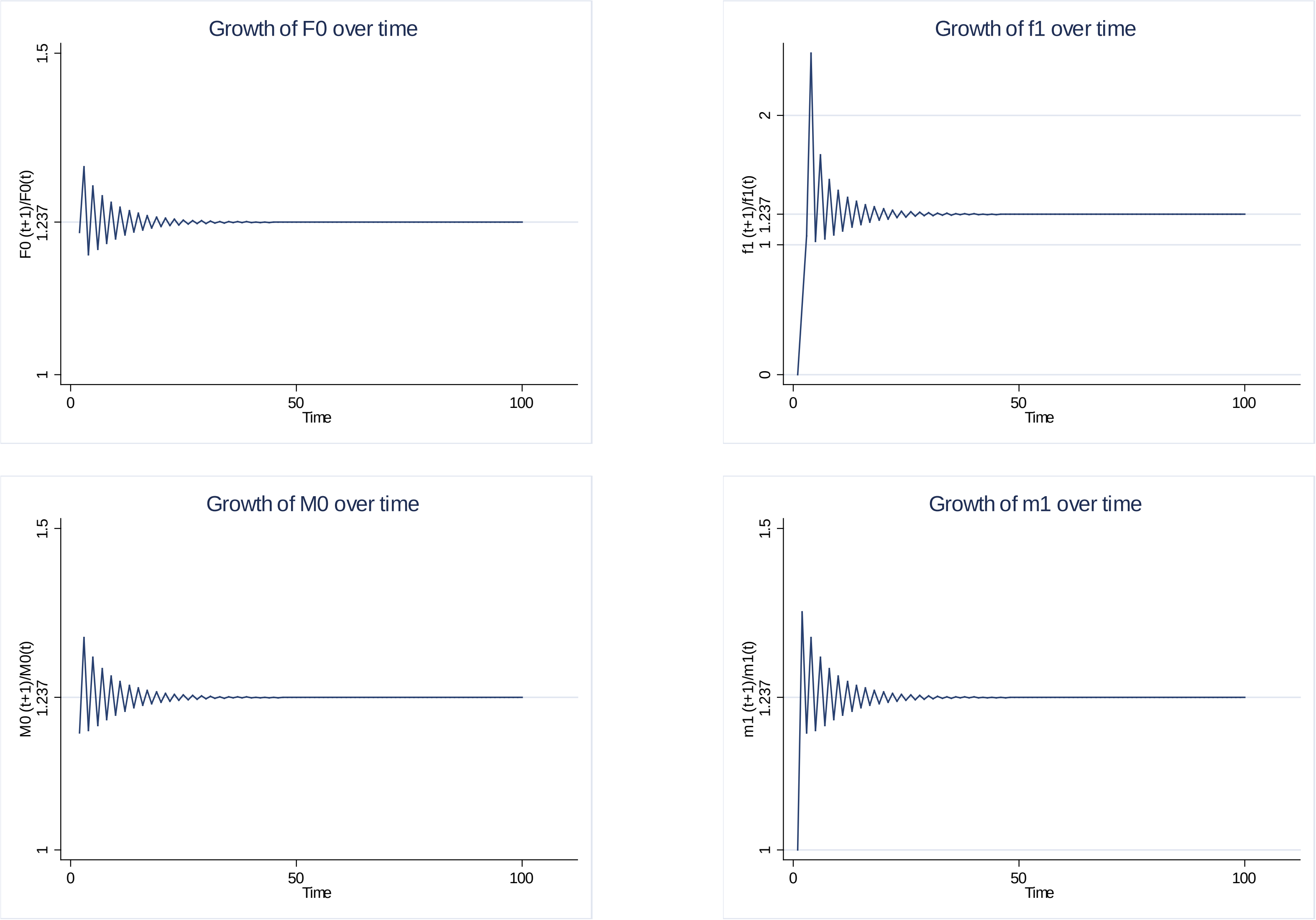

Table (a) contd.

The next two graphs demonstrate that in the long run equilibrium, $p_0 = 0.787$, $p_1 = 1$.