Preventing Public Input Competition

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Abstract

Increasing returns to scale may imply that public investment should be concentrated in one region of a federation. In the absence of side payments, non–cooperative behaviour may lead to inefficient dispersion of this public investment. If regional decision makers are "xenophobic", and do not value the earnings of non–resident inputs employed in the region, then this dispersion may be relatively easily prevented. Even without negotiation, the Nash equilibrium may involve public investment in only one region. With xenophobic decision makers, however, negotiation will result in the region without public investment compensating the other, more developed region to invest more.

1 Introduction

A commonplace of political rhetoric is the notion that regional development requires investment in public infrastructure. "Ensuring that Michigan has sufficient infrastructure to support economic development is essential to competing in the 21st Century economy" begins a typical press release press release[1] describing a programme funding public investment[2]. The rhetoric does have some evidence behind it. While there has been considerable debate about how productive public investment is[3], it certainly does appear that firms' location decisions are influenced by regional infrastructure levels.[4].

But the rhetoric praising public infrastructure investment often emphasizes its role in attracting mobile resources from other jurisdictions. Much of the empirical literature is consistent with this emphasis, showing a positive impact of public investment on private investment in a jurisdiction, or employment, or population. What tends to be ignored in the rhetoric is that one jurisdiction's gain may be another jurisdiction's loss. If regional policy makers believe that public investment attracts mobile factors, public input competition may be another manifestation of the race to the bottom.

The idea that one jurisdiction's public investment may harm other jurisdictions' economies is not a novel one. Boarnet (1998) shows how transportation improvements in one region may have negative spillovers, and his work has been cited as showing a possible specification problem in empirical work on the productivity of public invest-

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[1] url : http://www.michigan.org/medc/news/major/archive/combo.asp?ContentId=F33D9E99-E436-44D9-BB77-3A8D3727ED77&QueueId=2&ContentTypeId=7

[2] Two other examples from a vast quantity of press releases : the announcecement of a 2005 World Bank report
[url : http://web.worldbank.org/WBSITE/EXTERNAL/COUNTRIES/LACEXT/0,,
contentMDK:20631899 pagePK:146736 piPK:146830 theSitePK:258554,00.html] headlined "Latin America Needs Better Infrastructure to Compete with Asia", a Mumbai analyst noting [url : http://inhome.rediff.com/money/2005/sep/23bweek.htm] that "Compared with China's infrastructure spending of 20% of its gross domestic product, India spends just 6%. It needs to increase that to at least 10% to prime economic growth further."

[3] see Aschauer (1989) for an influential study finding high productivity ; Holtz–Eakin (1994) is one of many papers casting some doubt on the magnitude of this effect.

[4] see, for example, Demetriades and Mamunea (2000), Fisher (1997), Holl (2004) or Mark, McGuire and Papke (2000)

ment. Keen and Marchand (1997) show that tax competition considerations will bias the composition of jurisdictions' public expenditure, in favour of productive inputs at the expense of consumption goods. Martin and Rogers (1995), Bougheas, Demetriades and Morgenroth (2003), and Dupont and Martin (2006) all use variants of the "core–periphery", Dixit–Stiglitz continuum–of–goods model to show the incentives for over–investment when jurisdictions decide public investment non–cooperatively. Bucovetsky (2005) uses a more rudimentary model to show the inefficiency of Nash equilibrium when jurisdictions' governments' investment decisions are made non–cooperatively.

If public input competition is a problem, what should be done about it? Federal governments may already be taking measures to counteract regional incentives to over–invest, whether or not the measures were intended to do so. Johnson (1988) and Smart (1998) showed the perverse incentives federal transfer systems may have on spending decisions of lower level governments. Is some sort of federal transfer scheme needed as a corrective instrument for the negative spilllovers due to uncoordinated public investment decisions?

The efficiency of non–cooperative public infrastructure investment is analyzed in this paper, using a simplified version of the model presented in Bucovetsky (2005). In this model, scale economies imply that diversification of public infrastructure among regions will be inefficient. So the nature of the efficient allocation is **not** the focus of the paper. Here an efficient allocation has infrastructure concentrated in one region. One question addressed here is whether this specialization might emerge in equilibrium, or whether some transfer system might be needed to induce it.

A second question addressed here is the importance of regional policy–makers' goals. Many politicians appear to want to maximize total employment in their jurisdiction, or total investment there, or the value of output. Quite apart from concerns over the environment or congestion, residents of a region might want to take account of the costs of development. They might want to subtract off the returns of non–resident investors from the total benefits of new investment. Delegating authority to a politician who ignores these costs may lead to more aggressive public investment strategies. How sensitive is the equilibrium outcome to the choice of decision maker?

A third question addressed here is the potential for negotiation among governments. In much of the fiscal competition literature, individual governments are assumed to behave non–cooperatively. But they can, and do, negotiate with each other.[5] Negotiation may not be costless. In particular, governments' public investment choices may be influenced by how the level of investment affects their bargaining power in subsequent negotiation. To what extent might a "hold–up" problem emerge, in which a regional policy maker might want to race to get public infrastructure bolted down in her own jurisdiction?

Finally, the structure of the transfers which might result from negotiation is considered briefly. In many federations, inter–governmental transfer systems have been approved unanimously by all constituent regions. Would negotiation over public infrastructure yield a system of side–payments which resemble existing transfer systems?

In the model used here, public infrastructure is useful only in one sector. But it must be assumed that economies of scale are large in that sector, and that much of the input used in the sector is very mobile. More precisely, it is assumed that the sector exhibits increasing returns to scale with respect to the public input, and a perfectly mobile input, combined. The restriction of scale economies to a single sector may help to focus attention on what may be the difficult problems in organizing a federation. A considerable volume of literature on fiscal competition assumes identical regions, and a constant–returns–to–scale technology. In such a setting, non–cooperative behaviour by regional governments often may lead to an inefficient outcome. The question then does arise : "why don't the regions just agree on a Pareto improvement?". Typically, no transfers need to be made, when regions are identical. Regions will have incentives to renege on a negotiated agreement, but some of the policies which they choose do seem verifiable.

Scale economies in public investment need not lead to asymmetry, either in the Nash equilibrium or in the efficient solution. Some variants of the "new economic geography" have increasing returns to scale, and may have symmetry. With a continuum of differentiated goods, the optimum may involve having each region produce a different set of goods, with each region producing the same number of goods. The symmetry of this outcome means that this optimum should be relatively easy to negotiate. In contrast, in the model used here, efficiency requires specialization in public investment in a single region. That may be a more difficult negotiation problem.

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[5] a point recognized in Persson and Tabellini (1992), and Lülfesmann (2002), for example

Negotiation need not be restricted to "horizontal" bargaining among regional governments. But it should be recognized that most central governments' legislatures are comprised of representatives from different districts. Central government policy may be determined by bargaining among these representatives, as in Besley and Coate (2003) for example. Such bargaining over distributive public spending tends to result in geographical dispersion of public investment, as a result of log–rolling. The more difficult task is concentrating some project in a single region, especially when legislators are prevented (by constitutional rule, or by precedent) from making side payments to "losing" districts. It seems much more common for one (somewhat) sovereign regional government to agree to transfers to another region, than for legislators to approve cash transfers to one district in a unitary state in exchange for public investment in another. So again, it is the asymmetric outcomes which will be most difficult to sustain.

In summary, there are categories of public investment in which a nation's income will be maximized by the construction of one big project in one region. Decentralization may give regions the power to negotiate side payments, a power which is less likely to arise in a democratic centralized nation. The questions addressed here are whether such side payments must be negotiated, with a brief consideration of the form they might take.

2 Description of the Model

It will be assumed throughout that a federation is divided between 2 regions, which are identical in population, technology and resource endowment. Each region is endowed with a fixed supply of a specialized factor, which can be used only in one particular industry. This specialized factor is perfectly mobile between regions. In the absence of any regional source–based taxes, or quantity restrictions, the factor will move so that its return is the same in each region. However, the industry which uses this mobile factor uses as well a public input. This public input is completely immobile among regions. It will also be assumed that this public input is necessary for the industry ; a region which does not invest in the public input will attract none of the mobile factor, since the marginal product there of the mobile factor would be 0.

Most crucially, there are increasing returns to scale in these two inputs together. I will assume a Cobb–Douglas technology, in which the (constant) elasticity of the output of this industry, with respect to the public input and the mobile factor together, exceeds 1. Of course, this formulation requires high elasticities for both of these inputs. If, for example, one takes an estimate of the elasticity of output with respect to public input of about 1/3 consistent with Aschauer's (perhaps rather high) estimates, then perfectly mobile inputs would have to account for at least 2/3 of the private costs of production.

In defence of the plausibility of this assumption, note that it is a particular sector of the economy that is being modelled here. While the elasticity of overall output with respect to public investment may not be very high, there may be some (important) industries where output is much more sensitive. Second, barriers to mobility within some (homogeneous?) federations may be small enough that it is reasonable to assume that most of the purchased inputs, capital and labour, are relatively mobile. Finally, the importance of public inputs in attracting factors can be motivated by assuming that there some economies of scale, internal to an industry within a region, but external to the constituent firms. So public infrastructure may not be that important for the industry, but it may help attract workers, and the presence of specialized workers may attract other workers. Here these economies are not modelled explicitly. Bucovetsky (2005) does allow for the elasticity of output with respect to the mobile factor to exceed the factor's share in output. Here that possibility is dropped, in the interests of tractability, but the results would not change much if such externalities were allowed.

If regions choose different levels of public input, then there will be factor movement in equilibrium. Regions with a relatively high share of total public investment will be net importers of the mobile factor.

Who cares about the return received by units of the mobile factor which are employed in region 1, but which originated in region 2? If the mobile input is capital, the usual practice is to assume that this return is a benefit only to region 2. In this case regional decision makers want to maximize the value of output, net of payments made to the imported mobile factor. For the most part, that is the assumption I will make here : earnings of the mobile factor are "credited" by residence, not source.

If the mobile factor is labour, this assumption implies that the earnings of immigrants are not considered part of a region's payoff. This would not be the case, of course, if suppliers of the mobile factor moved to the region, settled there, and acquired the same voting rights and political power as existing residents. But suppliers of the mobile factor may be temporary “guest workers”. A recent, related literature has explored the implications of treatment of new immigrants in achieving an efficient international allocation of population. Sinn (2005) has pointed out the effects of “selectively delayed integration”, coupled with wage subsidies, as a means of protecting domestic redistribution programmes from the pressures introduced by free international mobility of low–income workers. Richter (2004) and Weichenreider and Busch (2007) as well analyze the implications of delayed integration. If integration of mobile labour is delayed indefinitely, the earnings of these workers should be “credited” in the country of origin, not in the country in which they work.

The model would apply as well if the “mobile” factor did not move at all, but was employed in producing a specialized intermediate input, and this intermediate input was then shipped (costlessly) to regions with higher investment in the complementary public input. In all of these cases, the maximand of policy makers in region 1 would not include any earnings to suppliers of the mobile factor who originated in region 2 : these earnings would be subtracted from the gross return to public investment in the anlysis of the benefits of that investment.

The opposite assumption was made in Bucovetsky (2005). There each region’s policy makers wished to maximize the total value of the sector in their region, minus the cost of public input provided. For concreteness, I will refer to policy makers who take this latter view (source based) as “boosters”, and (residence based) politicians who do not include returns to foreign–owned inputs in their maximand as “xenophobes”.

Of course, politicians in one region may be boosters, and politicians in the other region xenophobes. A political outcome of this type might be very useful in ensuring that infrastructure is concentrated in the first region. However, the coexistence of these two types of decision maker leads to some disturbing analytic problems. In this case, a movement of resources from the second region to the first might reduce aggregate income in the federation, while at the same time it raised the payoff of decision makers in each region. In the model used here, there is a simple, natural measure of overall welfare for the whole federation. For the most part, I restrict attention to environments in which the sum of welfare in individual regions must equal this overall aggregate welfare.

The formal model is presented immediately below. However, some of the implications of the assumption of increasing returns (and of Cobb–Douglas form) can be seen from Figure 1. In this figure, the payoff to (xenophobic) region 2 is graphed, as a function of its own public investment G_2. This payoff depends as well on the other region's choice G_1 of public investment. It declines with region 2's own investment, at small levels of G_2, but then increases with G_2 at larger levels, and then decreases. There are thus 2 possible optimal policies : $G_2 = 0$, or the (unique) local maximum at which $G_2 > 0$. Increases in G_1 shift up the curve, if (and only if) $G_2 < G_1$. But they shift it up more at $G_2 = 0$ than at the other local maximum. In the figure, region 2's best response to region 1's choice of public investment is to undertake a positive level of public investment if G_1 is low, but not to undertake any investment at all if G_1 is high.

Using the terminology of Eaton (2004), in this figure the regions' public investment levels are "plain complements" if $G_2 < G_1$, but are "strategic substitutes". If efficiency requires that region 2 not undertake any public investment, then increases in G_1 seem to be helpful : they lower region 2's preferred level of investment, should it be positive, and they lead, eventually, to the region deciding not to invest at all. On the other hand, they increase region 2's payoff (if $G_2 < G_1$), which means that they might also improve region 2's bargaining position, depending on their direct effect on region 1's own payoff. Unlike many situations in oligopoly, or in labour relations, here preemptive investment may not be an attractive strategy for region 1, should it be able to commit early to a public investment level.

3 Model Details

If region i has a public input level of G_i, and if L_i mobile workers[6] are employed there, then the value of output produced in region i is assumed to be

$$F(G_i, L_i) = G_i^g L_i^a \tag{1}$$

where a and g are positive constants, with

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[6] I will refer to the mobile factor as labour, and to its return as its wage.

$$0 < a < 1 \quad ; \quad 0 < g < 1 \quad ; \quad a + g > 1 \tag{2}$$

so that there are increasing returns to scale in the two inputs together but decreasing returns to scale in each input alone. The assumption that $a + g > 1$ implies that it will be inefficient to diversify public investment among different regions.

Two measures of the strength of scale economies help determine the outcome in this model. The measures are defined as

$$b_X \equiv g + a2^{a-1} - 1 \tag{3}$$

$$b_B \equiv 2g + (2 - g)a - 2 \tag{4}$$

What will matter is not the magnitudes of b_X or b_B, but their signs. Both b_X and b_B increase with the degree of scale economies ; each is an increasing function of a and of g. If there are constant returns to scale in both inputs — $a + g = 1$ — then both b_X and b_B are negative. Moreover, it is more likely that b_X is positive than b_B.

Lemma 1 *Under assumption (2), if* $b_B > 0$ *then* $b_X > 0$.

The proofs of this lemma and of most of the other derivations are contained in the appendix.

There are 2 regions, each with the same number of workers initially resident. There are $\bar{L}$ workers in total, $\frac{\bar{L}}{2}$ from each region. Workers are paid the value of their marginal product, so that the wage of mobile workers employed in region i is

$$w_i = aG_i^g L_i^{a-1} \tag{5}$$

if $G_i > 0$. Workers are perfectly mobile among regions, so that w_j is the same in each region in which there is a positive level of public input. This mobility implies that the number of workers employed in region i will be

$$L_i = s_i \bar{L} \tag{6}$$

if at least one of the public investment levels G_j is positive , where

$$s_i \equiv \frac{G_i^\gamma}{G_1^\gamma + G_2^\gamma} \tag{7}$$

and where the parameter γ is defined as

$$\gamma = \frac{g}{1-a} > 1 \tag{8}$$

so that the value of output produced in region i will be

$$G_i^g (s_i \bar{L})^a$$

The share of workers in region i is an increasing function of the region's level of public investment, and a decreasing function of the public investment in the other region.

$$\frac{\partial s_i}{\partial G_i} = \gamma \frac{s_i(1-s_i)}{G_i} \tag{9}$$

and

$$\frac{\partial s_i}{\partial G_k} = -\gamma \frac{s_i s_k}{G_k} \qquad i \neq k \tag{10}$$

The cost of the public input is c per unit. The maximand of a xenophobic government is

$$\pi_i^X = (1-a)G_i^g (s_i \bar{L})^a + aG^g (s_i \bar{L})^{a-1} \frac{\bar{L}}{2} - cG_i \tag{11}$$

if $G_i > 0$: the first term in the above expression is the value of output, net of wage payments, the second term is wage earnings of people originally resident in the region, and the third term is the cost of the public input. In contrast, the maximand of a booster government would be

$$\pi_i^B = G_i^g (s_i \bar{L}_i)^a - cG_i \tag{12}$$

If $L_1 = L_2 = \frac{\bar{L}}{2}$, then $\pi_i^B = \pi_i^X$, but otherwise the two payoffs are different.

Expression (11) can also be written

$$\pi_i^X = G_i^g (s_i \bar{L})^a [(1-a) + \frac{a}{2s_i}] - cG_i \tag{13}$$

From equation (13), the derivative of a region's payoff with respect to its own public investment level is

$$\frac{\partial \pi_i^X}{\partial G_i} = G_i^{g-1}(s_i \bar{L})^a([(1-a) + \frac{a}{2s_i}][g + a\gamma(1-s_i)] - \frac{a}{2s_i}\gamma(1-s_i)) - c \tag{14}$$

The definition $\gamma = g/(1-a)$ implies that this expression can be written

$$\frac{\partial \pi_i^X}{\partial G_i} = gG_i^{g-1}(s_i \bar{L})^a[1 + a(\frac{1}{2} - s_i)] - c \tag{15}$$

As $G_i \to 0$, the above derivative must be negative. When regions have very little public input, then marginal increases in G_i do not increase the labour supply much ; the elasticity of s_i with respect to G_i approaches 0 as $s_i \to 0$. This weak effect of public input in attracting labour makes the increased net income smaller than the cost of investment. So, if $G_2 > 0$, there must be a local maximum to xenophobic region 1's payoff at $G_1 = 0$. However, as the next section discusses, this local maximum need not be a global best response.

4 Nash Equilibria to the Simultaneous–Move Game

If $G_1 = G_2$, then each region would employ half of the stock of the mobile factor. Equation (15) implies that a public investment pattern $G_1 = G_2 = G^E$ can be a Nash equilibrium (when both regions are xenophobic) only if

$$g[G^E]^{g-1}[\frac{\bar{L}}{2}]^a = c \tag{16}$$

There is a unique value for G^E which satisfies this condition (16). By construction, $G_1 = G^E$ satisfies the first–order conditions for maximization of π_1^X when $G_2 = G^E$ (and vice versa). But for $G_1 = G_2 = G^E$ to be a Nash equilibrium, it must be the case that the second order conditions are satisfied as well, that $G_1 = G^E$ is a local maximum, and not a local maximum. Moreover, it must be a global maximum. Figure

1 shows that the payoff to a region need not be single–peaked as a function of its own public investment level ; expression (15) shows that this payoff must be decreasing with G_1 at $G_1 = 0$.

In fact, the payoff function of a xenophobic region must have the shape illustrated in figure 1 ; it can have at most one local maximum (other than no investment at all).

Lemma 2 *Holding the other region's public investment fixed, there can be at most one local maximum to region i's payoff* π_i^X *as a function of* G_i*, other than* $G_i = 0$.

This lemma implies that, if $G_1 = G^E$ is a local maximum to region 1's payoff (when $G_2 = G^E$), then the only other possible best response for the region is to undertake no public investment at all.

The payoff to a region when $G_1 = G_2 = G^E$ is

$$\pi^{XE} = \frac{1-g}{g} c G^E \tag{17}$$

from the first–order condition (16). If, instead, region 1 chose not to do any public investment at all then region 2 would have all $\bar{L}$ of the mobile factor employed there, so that the return to the mobile factor would be

$$w = aG^g \bar{L}^{a-1}$$

which means that total wage earnings of people originally resident in region 1 would be $w\bar{L}/2$, or

$$\hat{\pi}_1^X = a[\frac{1}{2}]^{1-a}[G^E]^{g-1}[\frac{\bar{L}}{2}]^a \tag{18}$$

From equations (17) and (18), this deviation from $G_1 = G^E$ to $G_1 = 0$ will be worthwhile if $\hat{\pi}_1^X > \pi^{XE}$, which will be true if and only if $b_X > 0$. So $b_X \leq 0$ is a necessary condition for the existence of a symmetric Nash equilibrium when both regions' decision makers are xenophobic. As will be shown subsequently, this condition is both necessary and sufficient.

Consider next the level of investment which a region would choose unilaterally, if the other region chose not to invest at all in the public input. If region 1 had public investment of G_1, and the other region had $G_2 = 0$, then region 1 would receive a payoff of

$$\pi_1^X = G_1^g \bar{L}^a [1 - a + \frac{a}{2}] \tag{19}$$

This payoff is a concave function of region 1's level of public input, and is maximized at an investment level G^S such that

$$g(G^S)^{g-1} \bar{L}^a [1 - a + \frac{a}{2}] - c = 0 \tag{20}$$

Comparing equations (20) and (16), $G^S > G^E$ if and only if $2(1 - a) + a - 2^{1-a} > 0$, which must be true if $a < 1$.

Given the scale economies, the globally efficient outcome, that which maximizes the sum of regions' payoffs, involves concentration of investment in a single region. The optimal level of public investment G^* to undertake in that single region is defined by the first–order condition

$$g(G^*)^{g-1} \bar{L}^a - c = 0 \tag{21}$$

Since a xenophobic region acting unilaterally does not take into account the positive benefits its investment confers on other regions (by raising the wage paid to mobile workers from those regions), comparison of (21) and (20) shows that $G^S < G^*$. Summarizing these results,

Proposition 1 $G^E < G^S < G^*$

The investment levels $G_1 = G_2 = G^E$ will be a Nash equilibrium only if the second–order conditions for each region's maximization problem are satisfied. This will be the case only if returns to scale are not too high. If these second–order conditions do not hold, then there cannot be a Nash equilibrium with positive investment in both regions.

Lemma 3 *If the 2 regions are identical, if $G_1 = G_2 = G^E$ does not satisfy the second–order conditions for each region, then there will be no Nash equilibrium in which each (xenophobic) region undertakes a positive level of public investment.*

Moreover, even if $G_1 = G_2 = G^E$ is a Nash equilibrium, it may not be stable, in the sense that regions' reaction functions' slopes are less than 1 in absolute value.

Lemma 4 *i if $\frac{a}{1-a}\frac{g}{1-g} \geq 4$ then there is no symmetric Nash equilibrium*

ii if $2 \leq \frac{a}{1-a}\frac{g}{1-g} < 4$ then $G_1 = G_2 = G^E$ is a "local" Nash equilibrium, in that neither region can gain from a small change in its own public investment ; however, the Nash equilibrium is unstable, in that reaction functions have a slope greater than or equal to 1 in absolute value

iii if $\frac{a}{1-a}\frac{g}{1-g} < 2$ then there is a stable "local" Nash equilibrium ; no small unilateral deviation from $G_1 = G_2 = G^E$ can increase a region's payoff

An implication of Lemma 4 is

Proposition 2 *When regions' decision makers are xenophobic, and choose public investment levels simultaneously and non–cooperatively, then $G_1 = G_2 = G^E \equiv [\frac{g}{c}]^{1/(1-g)}[\frac{\bar{L}}{2}]^{a/(1-g)}$ will be a Nash equilibrium if and only if $b_X \leq 0$.*

An alternative possibility for equilibrium is an outcome in which one region chooses not to invest at all. In that case, the other region would choose a level of G^S, its best reaction to the first region's choice of zero investment. Such a pair $((0, G^S)$ or $(0, G^S))$ will constitute a Nash equilibrium if and only if $G_1 = 0$ is region 1's overall best response to $G_2 = G^S$. The following lemma helps establish conditions under which such an equilibrium might exist.

Lemma 5 *Let $G_1 = G^R(G_2)$ be the value of G_1 which satisfies region 1's first–order condition (15), when region 2's public input is G_2. Then*

$$\frac{\partial}{\partial G_2}[\pi_1^X(0, G_2) - \pi_1^X(G^R(G_2), G_2)] > 0$$

(where $\pi_1^X(G_1, G_2)$ denotes the payoff to region 1 when it chooses a public investment level of G_1 and region 2 chooses G_2).

This lemma can be used to show that there must be an equilibrium in which one region chooses not to invest, if there is no symmetric Nash equilibrium.

Proposition 3 *Either there is a symmetric Nash equilibrium, or there is an asymmetric equilibrium, in which one region chooses* $G_i = 0$ *and the other region chooses* $G_k = G^S$.

Proof. If $b_X \leq 0$, then Proposition 2 shows that there must exist a symmetric Nash equilibrium. If $b_X > 0$, then region 1's best response to region 2's choice of $G_2 = G^E$ is $G_1 = 0$; that is

$$\pi_1^X(0, G^E) > \pi_1^X(G^R(G^E), G^E)$$

if (and only if) $b_X > 0$. Proposition 1 implies that $G^S > G^E$ so that (from Lemma 5)

$$\pi_1^X(0, G^S) > \pi_1^X(G^R(G^S), G^S)$$

whenever $b_X > 0$, implying that $(G^S, 0)$ is a Nash equilibrium (as is $(G^S, 0)$). •

There must be some cut–off level of public investment for region 2 which "deters entry" by region 1. That is, there is some level of public investment G^D such that

$$\pi_1^X(G_2^R(G^D), G^D) = \pi_1^X(0, G^D)$$

Region 1's overall best response to G_2 will be positive if (and only if) $G_2 < G^D$. $G_1 = G_2 = G^E$ will be a Nash equilibrium if and only if $G^E < G^D$. $(G_1, G_2) = (G^S, 0)$ will be a Nash equilibrium if and only if $G^S \geq G^D$. If $G^E < G^D < G^S$, then both symmetric and asymmetric Nash equilibria exist.

Existence of other Nash equilibria (in which regions' public investment levels differ, but in which they both are positive) cannot be ruled out[7].

The outcomes may be different when regional decision makers are boosters. Nash equilibrium with this criterion for regional decision makers was analyzed in Bucovetsky (2005) ; there a necessary and sufficient condition for the existence of a symmetric Nash equilibrium is that $b_B \leq 0$.

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[7] although I have not found any in any of the calculated examples

Lemma 1 therefore implies that if a symmetric Nash equilibrium exists when decision makers are xenophobic, then a symmetric Nash equilibrium exists when decision makers are boosters.

Moreover, in each case, the common levels of public investment are the same in a symmetric Nash equilibrium, whether regions are xenophobic or boosters.

If $b_B > 0$, then there may be an asymmetric equilibrium when decision makers are boosters. However, now the region which chooses to invest will choose the efficient level G^* of investment, rather than $G^S < G^*$. In other words, the asymmetric equilibrium is better when regions are boosters : the region doing the investing internalizes fully the benefits of its investment.

But the asymmetric equilibrium is much less likely to arise when decision makers are boosters. As noted above, wasteful duplication is more likely under boosterism. But in addition, there is no analogue to Proposition 3 when both regions are boosters. There may not be a Nash equilibrium in pure strategies when booster governments play non–cooperatively and simultaneously. In fact, if $g < 0.7$ (which does not seem an unreasonable restriction), then there cannot be any asymmetric equilibria when regions are boosters[8]. If $b_B > 0$ and $g < 0.7$, there is no equilibrium at all (at least in pure strategies). Booster regions are more likely to react to high levels of investment in the other region by investing even more, in order to attract the mobile factor. This aggressive over–investment precludes the existence of equilibrium when scale economies are high.

Thus a Nash equilibrium always exists if both regions are xenophobic, but not if they are boosters. The Nash equilibria under xenophobia are never efficient, but they are less likely to involve wasteful diversification of public investment.

5 Sequential Moves

There are a few reasons to consider sequential decision–making by regions. First, federations are often formed by dissimilar regions. It is worthwhile considering the implications of one region being able to commit to public investment before the other.

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[8] as shown in figure 3 of Bucovetsky (2005)

Second, when $G^S > G^D$, there are multiple equilibria to the simultaneous–move non–cooperative game. Some pre–game communication, or other such complication, may be needed to select an outcome. Sequential moves may be a simple way of narrowing the range of possible outcomes.

Third, when regions' decision makers are boosters then there will be no Nash equilibrium in pure strategies when they move simultaneously, if $b_B > 0$. The implications of the different maximands can be considered when regions move sequentially.

Suppose then that region 1 moves prior to region 2.

Prior to proceeding, it may be useful to summarize some properties of the "reaction function" [9]

Lemma 6 *(i) The function* $G_2 = G^R(G_1)$ *has a positive slope if and only if* $s_2 > \frac{2+a}{2+2a} > \frac{3}{4}$.

(ii) The slope of the function $G_2 = G^R(G_1)$ *equals 0 at* $G_1 = 0$.

(iii) $G^R(G) > G$ *if and only if* $G < G^E$.

(iv) if $G_1 = G^E$, *then* $\frac{\partial G^R}{\partial G_1} = -\frac{a\gamma}{4(1-g)-a\gamma}$

Region 1's payoff can be written as

$$F(G_1, L_1) - w_1(\frac{\bar{L}}{2} - L_1) - cG_1$$

so that the fact that labour is paid the value of its marginal product implies that the change in region 1's payoff, in response to a slight increase in the level of its own public investment, is

$$[F_G - c] - [\frac{\bar{L}}{2} - L_1]\frac{dw}{dG_1} \tag{22}$$

where the term dw/dG_1 here denotes the overall effect on the wage, taking into account region 2's reaction, and the resulting reallocation of the mobile factor.

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[9] $G^R(G_1)$ is defined as the (unique) level of investment G_2 for region 2 which satisfies the first–order condition for optimality when region 1 chooses an invetment level G_1; thus it is the overall best reaction for region 2 only when it provides a better payoff than not investing at all (that is, only when $G_1 < G^D$)

The strategic effect of moving first operates through this second term. But the terms of trade matter only when there are non–negative net imports or exports. At $G_1 = G^E$, by definition region 2's best reaction will be $G_2 = G^E$ (if $G^E < G^D$). So this terms of trade effect disappears at the symmetric Nash equilibrium. Expression (22) will be zero at $G_1 = G^E$.

Does that mean that the outcome when regions move sequentially will still be the symmetric Nash equilibrium? This will be true only if $G_1 = G^E$ is a local maximum for region 1's payoff, rather than a local minimum.

If $\pi_1^{XS}(G_1)$ denotes the payoff to region 1 when it moves first, and when region 2 chooses a public investment level of $G^R(G_1)$, then

$$\frac{\partial \pi_1^{XS}}{\partial G_1} = F_G^1 - c + \frac{\partial w}{\partial G_1}[\frac{\bar{L}}{2} - L_1] + \frac{\partial w}{\partial L_2}\frac{\partial G^R}{\partial G_1}[\frac{\bar{L}}{2} - L_1] \tag{23}$$

where $F^1(G_1, L_1)$ is the production function in region 1. Differentiating yet again

$$\frac{\partial^2 \pi_1^{XS}}{\partial {G_1}^2} = F_{GG}^1 - \frac{\partial w}{\partial G_2}\frac{\partial G^R}{\partial G_1}\frac{\partial L_1}{\partial G_1}(1 - \frac{\partial G^R}{\partial G_1}) \tag{24}$$

at $G_1 = G^E$. This expression can be reduced to

$$\frac{\partial^2 \pi_1^{XS}}{\partial {G_1}^2} = \frac{F_G^1}{G_1}[-(1-g) + |\frac{\partial G^R}{\partial G_1}|\frac{a}{2}(1 + |\frac{\partial G^R}{\partial G_1}|)] \tag{25}$$

From part (*iv*) of Lemma 6, when $G_1 = G_2 = G^E$,

$$\frac{\partial G^R}{\partial G_1} = -\frac{a\gamma}{4(1-g) - a\gamma}$$

where the expression in the denominator must be positive if $G_1 = G_2 = G^E$ is a Nash equilibrium.

Therefore, the expression in square brackets in equation (25) is proportional to

$$-[4(1-g) - a\gamma]^2 + 2a^2\gamma \tag{26}$$

If economies of scale are minimal, so that $a + g = 1$, then expression (26) must be negative, so that $\frac{\partial^2 \pi_1^{XS}}{\partial {G_1}^2} < 0$ at $G_1 = G^E$. On the other hand, if the reaction function has a slope of -1, then (25) shows that $\frac{\partial^2 \pi_1^{XS}}{\partial {G_1}^2} > 0$.

That is, a symmetric Nash equilibrium may also be the Stackelberg equilibrium, if scale economies are small enough. But if they are sufficiently large, it will not. Figures 2 and 3 show the two possibilities. In the first figure $a = 0.7$ and $g = 0.4$, whereas in the second $a = 0.72$ and $g = 0.3$.

Whatever the actual choice of public investment by the first mover, however, the following result must hold :

Proposition 4 *Both (xenophobic) regions do at least as well when they make their public investment choices sequentially as they would in a symmetric Nash equilibrium to the simultaneous–move game.*

Proof. The region which moves first must do at least as well ; it always has the option of choosing $G_1 = G^E$ in the sequential game, and doing exactly as well as it did in the symmetric Nash equilibrium to the simultaneous–move game.

But region 2's payoff, should it choose its best positive level of public investment, is an increasing function of G_1 if and only $G_1 > G_2$. Thus its payoff from the pair $(G_1, G^R(G_1))$ is minimized at $G_1 = G^E$. Of course, it might choose to set $G_2 = 0$ in the equilibrium to the sequential–move game, but it would only choose to do so if $\pi_2^X(G_1, 0) \geq \pi_2^X(G_1, G^R(G_1)) \geq \pi_2^X(G^E, G^E)$. •

"Entry deterrence" may be an outcome here. If $G^D > G^E$, then region 1's payoff, as first mover, would jump discontinuously at $G_1 = G^D$, since π_1^X is a decreasing function of G_2 when $G_1 > G_2$.

But if scale economies are large enough, then being deterred is better than deterring. Note first that there may be a higher payoff here to being the low–investment region in an asymmetric simultaneous–move Nash equilibrium, if scale economies are large enough.

Lemma 7 *At* $G_1 = G^S, G_2 = 0$, $\pi_1^X > \pi_2^X$ *if and only if* $b_B < 0$.

As mentioned in the previous section, $b_B < 0$ also implies that there must be a symmetric Nash equilibrium to the simultaneous–move game.

If $b_B > 0$, then it is better to be the low–investment region. Because of this, the first mover must choose a lower level of investment.

Proposition 5 *If $b_B > 0$, then $G_1 < G_2$ when regions move sequentially.*

Choosing no public investment investment at all may be an attractive strategy for a xenophobic first mover, if scale economies are large. Such a choice forces the second region to choose $G_2 = G^S$. I have not been able to establish that $G_1 = 0$ must be the best strategy for region 1 when $b_B > 0$, but it must be "locally" the best strategy, in the following sense.

Proposition 6 *If $b_B > 0$, then a small increase in G_1 from $G_1 = 0$ must lower xenophobic region 1's payoff, when regions move sequentially.*

Proposition 5 shows that, not surprisingly, it matters a lot how regions' decision makers view income earned by the mobile factor. When they are xenophobic, and care only about income earned by the region's natives, this proposition shows that they exploit their first–mover advantage by forcing public investment to take place elsewhere, if scale economies are large. In contrast, if they are boosters, trying to maximize the income of all those employed in the region, then they try very hard to attract the mobile factor. In this case, if scale economies are large ($b_B > 0$), the sequential equilibrium must involve region 1 choosing a very large initial level of public investment, much larger than G^*, in order to deter region 2 from doing any public investment at all.

6 Negotiation among Xenophobic Governments

When non–cooperative behaviour leads to inefficient outcomes, it is natural to expect players to try and negotiate a better outcome.

In this section, a very simplistic view of negotiation is taken : I assume that regional governments are able to commit, to both investment levels and to interregional transfers.

I also assume a very simple sequence of events. Regions 1 and 2 begin negotiation at a time at which region 1 already may have some level G_1^0 of public investment in place ; region 2 has no initial investment. The regions then negotiate. If they do not reach an agreement, then they choose their investment levels non–cooperatively and simultaneously.

Note that there already is an asymmetry at the beginning of negotiation, since only region 1 has already undertaken any investment. I allow for pre–existing investment in one region for 3 reasons. First of all, this sequence of events may describe very crudely a process of globalization : initially region 1 is an isolated developed region, making its own public investment choice with a fixed supply of labour ; subsequently, region 2 becomes more developed, making it capable of undertaking public infrastructure investment, and also capable of supplying skilled labour to other regions, as barriers to mobility fall. Secondly, allowing for some pre–existing level of public infrastructure in one region leads to a simple device to analyze the strategic incentives for public investment. The effects of changes in G_1^0 on the results of negotiation can be examined.[10]. Thirdly, there do appear to be persistent inequalities in development levels within federations. Assuming that region 1 is the more developed region provides a vehicle to look at transfers between regions in this sort of federation.

I assume to the outcome of negotiation is the axiomatic Nash bargaining solution, with equal weights on each region. The particular form of bargaining solution is not crucial : the two crucial properties are that the outcome of negotiation be efficient ; and that a region's payoff (after negotiation) be an increasing function of its own disagreement payoff (and a decreasing function of the other region's disagreement payoff).

The disagreement outcome is just the Nash equilibrium to the simultaneous–move game analyzed in section 4 — except for the fact that region 1 already has an investment level G_1^0 in place, so that its payoff from some outcome (G_1, G_2) (with $G_1 \geq G_1^0$) is $\pi_1^X(G_1, G_2) + cG_1^0$.

Now the Nash equilibrium to the simultaneous–move bargaining game may not be unique. If $G^E \leq G^D \leq G^S$, then there must be at least 3 equilibria : (G^E, G^E), $(G^S, 0)$ and $(0, G^S)$. If $G^D < G^E$, then there are 2, $(G^S, 0$ and $(0, G^S)$. Even if $G^D > G^S$, there may exist asymmetric Nash equilibria in which both regions choose positive investment levels.

The presence of pre–existing investment in region 1 may eliminate some equilibria : obviously the outcome $(0, G^S)$ cannot arise if $G_1^0 > 0$. But it may also introduce new

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[10] and denying region 2 the opportunity to invest prior to negotiation avoids some of the complications of solving for a Nash equilibrium to a two–stage game, in which regions choose investment before and (potentially) after negotiation

equilibria, since $(G_1^0, G^R(G_1^0))$ will now be a Nash equilibrium if G_1^0 is a better reaction for region 1 to $G_2 = G^R(G_1^0)$ than any $G_1 > G_0^1$. (This certainly could be the case if G_1^0 were small, and $G^S > G^D$.)

Since region 1 may have already invested, I choose a very simple selection rule for equilibria in the event of disagreement : if negotiations fail, the outcome will be one which involves more investment in region 1. In particular, the disagreement outcome will be

$(G_1^0, 0)$ if $G_1^0 \geq \max(G^S, G^D)$

$(G^S, 0)$ if $G^S > \max(G^D, G_1^0)$

$(G_1^0, G^R(G_1^0))$ if $G^D > G^S > G_1^0 > G^E$

(G^E, G^E) if $G^D > G^S > G^E \geq G_1^0$

That is, I am selecting the asymmetric equilibrium $(G^S, 0)$ if it exists : this equilibrium Pareto dominates the symmetric equilibrium should both types exist. If this asymmetric equilibrium does not exist, I am selecting the symmetric equilibrium. And these selections are modified in the obvious way should region 1's initial investment already exceed its investment level in the selected equilibrium.

So suppose first that scale economies are small, so that $(G^S, 0)$ is not a Nash equilibrium. Then (G^E, G^E) will be the disagreement outcome (if region 1's initial investment does not exceed G^E). Since region 1 already has G_1^0 in place, its payoff in this disagreement outcome equals $\pi_1^X(G^E, G^E) + cG_1^0$.

The efficient outcome is to concentrate all investment in one region, which here must be region 1 if $G_1^0 > 0$, and the efficient amount to invest in region 1 is G^*, where $F_G(G^*, \bar{L}) = c$ [11]. If Π^* denotes the total payoff to both regions at this efficient outcome

$$\Pi^* = F(G^*, \bar{L}) - cG^*$$

then the combined payoff to both regions after negotiation is

$$\pi_1^{NX} + \pi_2^{NX} = \Pi^* + cG_1^0$$

Thus, in the axiomatic Nash bargaining solution, region 1's payoff is

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[11] since $G^E < G^*$, if $G_1^0 < G^E$, then further investment will be required in region 1 to reach this efficient level

$$\pi_1^{NX} = \frac{1}{2}[\Pi^* + cG_1^0] + \frac{1}{2}[cG_1^0] \tag{27}$$

where the second term on the right side of (27) is the difference between the two regions' disagreement payoffs.

Therefore, if scale economies are small, and if $0 < G_1^0 < G^E$, then each extra Euro "pre–invested" in infrastructure G_1^0 leads to a 1 euro increase in region 1's payoff. As long as $0 < G_1^0 < G^E$ there is neither strategic advantage nor disadvantage to committing to higher levels of investment.

If scale economies are small, it was shown in section 5 that G^E would maximize $\pi^X(G_1, G^R(G_1)$. In such a case, increasing G_1^0 above G^E would weaken region 1's strategic position : it would decrease its own disagreement payoff, and it would also increase region 2's, since $\pi_2^X(G_1, G^R(G_1))$ must increase with G_1 if (and only if) $G_1 > G^R(G_1)$.

Hence, when scale economies are small, strategic considerations imply that any initial level of investment up to G^E is equally good for region 1. If there actually is a lag, in which production takes place, between initial investment and negotiation, or if there is some chance that negotiation will not take place, then G^E seems the best choice, since G^E is the optimal level of public investment for an isolated region with a fixed supply $\bar{L}/2$ of labour.

Here, virtually by assumption, negotiation will prevent costly duplication of public investment. But the gains from locating investment efficiently are also small. In the limit, as $a+g \to 1$, $G^* = 2G^E$, and any investment profile (G_1, G_2) in which $G_1 + G_2 = G^* = 2G^E$ is efficient.

When (G^E, G^E) is the only Nash equilibrium to the simultaneous–move game played by xenophobic regions, regional decision makers' goals don't really matter for negotiation. (G^E, G^E) is the disagreement outcome whether they are both xenophobes, or both boosters. The outcome, and the transfers needed to sustain it, are the same in each case.

If $(G^S, 0)$ is a Nash equilibrium, it remains true that changes in G_1^0 have no strategic effect. When $0 < G_1^0 < G^S$, a 1 euro increase in G_1^0 will again increase the combined post–negotiation payoff by 1 euro, increase region 1's disagreement payoff by 1 euro, and have no effect on region 2's disagreement payoff. Increasing G_1^0 above G^S must harm region 1, since it reduces its own disagreement payoff, and increases region 2's disagreement payoff.

Thus, with xenophobic decision makers, there is no strategic role for any sort of preemptive over–investment, at least in the rudimentary framework for negotiation assumed here. When scale economies are high, so that $b_B > 0$ and $\pi_1^X(G^S, 0) < \pi_2^X(G^s, 0)$, a region would like to be the one not investing : but my convention that $G_1 = G^S$ in the disagreement outcome, regardless of what happens before negotiation, assumes away the possibility of becoming a "have–not" region strategically, in order to get better terms from negotiation.

The outcome of efficient negotiation is a binding agreement, in which region 1 commits to make the necessary investment $G^* - G_1^0$ to reach the efficient level, in which region 2 agrees not to invest, and in which transfers are made between regions.

What are the magnitude of the transfers? Region 1's payoff from the axiomatic Nash solution is

$$\frac{1}{2}[F(G^*, \bar{L} - cG^*] + cG_1^0 \tag{28}$$

provided that it makes the rational decision not to invest more than its disagreement–outcome level before negotiation begins.

Assuming that all public investment is financed from region 1's own tax revenue, its net payoff after negotiation can also be written

$$F(G^*, \bar{L}) - w^* \frac{\bar{L}}{2} - c(G^* - G_1^0) + T \tag{29}$$

where w^* is the wage rate at the efficient outcome, and T is the transfer region 1 receives from region 2. Equating expressions (28) and (29) implies that

$$T = \frac{1}{2}[cG^* + w^* \bar{L} - F(G^*, \bar{L}] \tag{30}$$

Optimality of G^* implies that $c = F_G(G^*, \bar{L})$. Thus increasing returns in the two inputs (public investment and the mobile factor) together are necessary and sufficient for the right side of equation (30) to be positive. If this is a federation of sovereign countries, so that each region funds its own public investment, transfers must flow from the "have–not" nation to the "have" nation to sustain the negotiated outcome. Put differently, any transfers from region 1 to region 2 must be less than the share of public investment in region 1 which is paid by region 2's taxpayers : the richer region must be a net fiscal beneficiary in order to be persuaded to undertake the efficient level of investment.

This direction of transfers can also be seen from another implication of increasing returns to scale : if the rich region undertakes the efficient level of investment, it must have a smaller payoff than the poor region, if both decision makers are xenophobic.

Lemma 8 $\pi_1^X(G^*, 0) < \pi_2^X(G^*, 0)$

7 Strategic Delegation

Not surprisingly, a booster government in region 2 may react more aggressively to existing public investment than a xenophobic government. If $b_B > 0$, region 2's best reaction to $G_1 = G^* > G^S$ must be to choose no public investment at all — if its decision maker is xenophobic. In contrast, Bucovetsky (2005) shows that a booster government's best reaction to $G_1 = G^*$ must be $G_2 > G^*$, provided that $g < 0.5$ and $b_B > 0$.

Suppose then that an efficient agreement has already been reached between the governments of regions 1 and 2, and the effiicient public investment plan $(G^*, 0)$ has already been implemented. Would regional governments want to renegotiate after the fact? Since the agreements discussed in the previous section involve net fiscal transfers from region 2 to region 1, the government of region 2 would certainly want to renege on its payments. This incentive to renege may explain why transfer payment rules are often made very difficult to amend, in the constitutions of federal states, or of federations of sovereign states, such as the EU.

However, most federal states do not constrain the rights of constituent jurisdictions to undertake public investment projects of their own. If region 2 has a xenophobic government, and if $G_1 = G^*$, region 2 would not want to exercise that right (if $b_B > 0$ so that $G_2 = 0$ was the best xenophobic reaction to $G_1 = G^*$). A booster government would want to exercise that right under those circumstances. Its best reaction to $G_1 = G^*$ is to undertake a level of investment greater than G^* (perhaps considerably greater), and thereby attract most of the mobile factor.

A booster government in region 2 would gain from deviation from the agreed outcome, even if it could not affect transfers. That gain does not necessarily imply a strategic advantage. Under the axiomatic Nash rule, the strategic advantage of region 2 is the difference between its disagreement payoff region 1's. If region 1 has a xenophobic government, then it may actually gain from region 2's booster government undertaking massive public investment. Consider the effect on π_1^X of increases in G_2, starting at the initial level of 0. As long as $G_2 < G_1$, further increases in G_2 harm region 1, since the wage rate increases, and it is a net importer of the mobile factor. But once G_2 equals G_1 further increases in G_2 will then start to increase π_1^X, since region is now a net exporter of the mobile factor.

However, it certainly may be the case that $\pi_2^B(G^*, G^{BR}(G^*)) - \pi_1^X(G^*, G^{BR}(G^*)) > \pi_1^B(G^*, 0) - \pi_2^X(G^*, 0)$, where $G^{BR}(\cdot)$ is the best reaction function of a booster government. Then, voters in region 2 would have an incentive for strategic delegation, to elect a booster government in order to re–open negotiations with region 1. The tendency of leaders of less–developed regions to seek grandiose projects, not justifiable by conventional cost–benefit analysis, may reflect rational behaviour by voters. The leaders' fixation with extravagant infrastructure projects may provide a credible threat to increase transfers from already–developed regions.

A serious analysis of the equilibrium strategic delegation will not be provided here. Instead a few more observations of tendencies and pitalls will be suggested.

First, when scale economies are large, a pure–strategy Nash equilibrium will not exist when booster regions choose investment levels simultaneously, meaning that the disagreement outcome when boosters negotiate may be difficult to ascertain. If they move sequentially, then an equilibrium will exist, and what it will involve (if scale economies are not to small) is the first mover picking a very large investment level, larger than the efficient level G^*, just high enough to deter entry by the second mover, if both are boosters.

So suppose that booster regions negotiated, with disagreement resulting in the sequential game. Region 1 would be forced to undertake a high, entry–deterring investment level in its first move, should there be disagreement. Region 1 would have a higher disagreement payoff than region 2. But if regions could commit only to transfers, not to investment levels, then it would actually have to undertake this high level of investment to prevent region 2 from threatening further escalation of investment. When regions move sequentially, and when they cannot commit to future investment levels, voters in region 1 might be better off electing a xenophobic government.

However, a problem emerges when a xenophobic government negotiates with a booster government. The sum of their payoffs does not equal the total net value of output in the two regions, $F(G_1, 0) - cG_1$ if investment is concentrated in region 1. If region 1 is run by boosters, and region 2 by xenophobes, wage payments to natives of region 2 employed in region 1 are counted twice. Negotiation between governments would lead to excessive investment if they agree to invest only in region 1 (run by the booster), or under–investment if they agree to invest only in region 2.

This inefficiency would not arise if delegation took place only after the efficient outcome had been agreed upon initially. If G^* has already been invested in region 1, and voters choose a xenophobic government there and a booster government for region 2, the irreversibility of investment would prevent negotiators from making a wrong decision.

Two further variations might also eliminate the inefficiency of decision makers of different type. Decision makers in a region might include both the income both of foreign nationals employed in their country, and the income of the country's natives employed elsewhere. This, after all, is the basis for American personal income taxation. And many people seem to take national pride both in natives who have gone on to success elsewhere, and in immigrants who have done well in their adopted country.

A less whimiscal variant is to count a person's income in the jurisdiction in which she permanently resides, but to let destination countries choose whether to grant permanent residence. So country 1 can choose whether to let guest workers from country 2 become permanent residents. On the other hand, I assume (somewhat realistically) that countries cannot prevent emigration. So if country 1 choose to grant permanent resident status, then if $G_1 > G_2$, income of the $\frac{\bar{L}}{2} - L_2$ natives of region 2 who are employed in region 1 will be included in region 1's income, not region 2's. Given this choice, then, both country's decision makers would be boosters if $G_1 > G_2$. This variant implies that voters in the region with the higher level of investment get to choose the common decision rule for both countries. Choosing to allow easy entitlement to permanent residence status then becomes a way for a country to commit to a more aggressive public investment policy.

8 Concluding Remarks

The main point of this paper is not exactly surprising : if industries in a region have to pay for the mobile factors they attract, then the region will be less likely to want to undertake public investment to attract those factors. Non–cooperative decision making leads to different sorts of inefficient outcome. If there are increasing returns to scale but these economies of scale are not too large, then the Nash equilibrium will be an inefficient dispersion of public investment. With bigger scale economies, public investment will be concentrated in one region, but that region will under–invest, since it neglects the benefits its own investment has on the return to imported factors.

Sovereign governments can, and do, negotiate, so that the inefficient Nash equilibrium may serve only as a constraint in the bargaining process. Here the less–developed region (the one in which no public investment takes place) has an incentive to bribe the other region to increase its public investment. So if the negotiated outcome is implemented by a system of intergovernmental transfers, then the transfers will flow from the less–developed region to the more–developed region.

Over–investment in order to attract mobile resources is not a threat here, since governments are assumed not to want to pay for the imported resources they use. However, matters change considerably if public investment attracts new residents, and these residents have an impact on the political process in the region to which they immigrate. Giving positive weight to the earnings of factors attracted by public investment makes regions' governments want to spend more aggressively. This enthusiasm for public investment may lead to a tendency to over–invest if regions behave non–cooperatively. But it may yield a strategic advantage in bargaining.

9 Proofs

9.1 Second–Order Conditions

Differentiation of the first–order condition with respect to G_i yields

$$\frac{\partial^2 \pi_i^X}{\partial {G_i}^2} = gG^{g-2}[1 + a(\frac{1}{2} - s_i)]^2 \Phi(s_i) \tag{31}$$

where

$$\Phi(s) \equiv (1 + a(\frac{1}{2} - s))(a\gamma(1 - s) - (1 - g)) - a\gamma s(1 - s) \tag{32}$$

Since the coefficient on $\Phi(s_i)$ in equation (31) is positive, the second–order conditions for optimality for G_i will be satisfied if and only if $\Phi(s_i) < 0$.

Now $\Phi(s)$ is a quadratic, which can be written

$$\Phi(s) = a\gamma(1+a)s^2 - [a\gamma(1+a) + a\gamma(1+\frac{a}{2}) + a(1-g)]s + [(1+\frac{a}{2})(a\gamma - (1-g)]] \tag{33}$$

The facts that $a > 1 - g$ and that $\gamma > 1$ imply that $\Phi(0) > 0$, and that $\Phi'(0) < 0$.

Also,

$$\Phi(\frac{1}{2}) = a\gamma(\frac{1}{4}) - (1 - g) \tag{34}$$

There can be a symmetric Nash equilibrium only if the expression on the right side of equation (34) is negative.

From (33),

$$\Phi'(s) = a[\gamma(1 + a)(\frac{1}{2} - (1 - s)) - (1 - g)] \tag{35}$$

9.2 Proofs of Lemmata and Propositions

Lemma 1 Under assumption (2), if $b_B > 0$ then $b_X > 0$.

Proof. Since $b_X = 0$ if and only if $a = 2^{-1a}(1 - g)$, and $b_B = 0$ if and only if $g(1-\frac{a}{2}) = 1-a$, then it is sufficient to show that $a > 2^{1-a}(1-g)$ when $g(1-\frac{a}{2}) = 1-a$.

If $g(1 - \frac{a}{2}) = 1 - a$, then

$$1 - g = \frac{a}{2 - a} \tag{36}$$

Substituting for g from (36), $a > 2^{1-a}(1-g)$ if and only if

$$a - 2(2^{-a})\frac{a}{2-a} > 0$$

or

$$2 - a - 2(2^{-a} > 0 \tag{37}$$

The left side of inequality (37) equals 0 when $a = 0$, and it equals 0 when $a = 1$. It also is a concave function of a, so that (37) holds whenever $0 < a < 1$, proving the lemma.
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Lemma 2 Holding the other region's public investment fixed, there can be at most one local maximum to region i's payoff π_i^X as a function of G_i, other than $G_i = 0$.

Proof. From equation (15), the second derivative $\frac{\partial^2 \pi_i^X}{\partial {G_i}^2}$ is proportional to

$$\Psi(s_i) = -(1-g)[1 + a(\frac{1}{2} - s_i)] + a\gamma(1 - s_i)[1 + \frac{a}{2} - (1+a)s_i]$$

The function $\Psi(s_i)$ is a quadratic, so has at most 2 roots. That means that, holding fixed the other G_j, there can be at most 2 values of $G_i > 0$ for which

$$\frac{\partial^2 \pi_i^X}{\partial {G_i}^2} = 0$$

Suppose now that there are 2 distinct local maxima to π_i^X. Recall that $\frac{\partial \pi_i^X}{\partial G_i} < 0$ at $G_i = 0$. Therefore, if there were 2 local maxima, then there would have to be at least 2 local minima (1 between $G_i = 0$ and the first local maximum, and the other between the 2 local maxima). With 4 local extrema, therefore $\frac{\partial^2 \pi_i^X}{\partial {G_i}^2}$ would have to change signs at least 3 times, contradicting the fact that $\Psi(s_i)$ is a quadratic. •

Lemma 3 If all regions are identical, if $G_1 = G_2 = G^E$ does not satisfy the second–order conditions for each region, then there will be no Nash equilibrium in which each region undertakes a positive level of public investment.

Proof. Equation (35) implies that $\Phi'(\frac{1}{2}) < 0$. Since $\Phi(s)$ is a quadratic, since $\Phi''(s) > 0$, and since $\Phi(0) > 0$, therefore, if $\Phi(\frac{1}{2}) \geq 0$, then it must be true that $\Phi(s) > 0$ for all $0 < s < \frac{1}{2}$. Suppose that $G_1 = G_2 = G^E$ did not satisfy the second–order conditions, which is equivalent to $\Phi(\frac{1}{2}) \geq 0$. If there were any other (G_1, G_2) which satisfied the first–order conditions for a Nash equilibrium (with each G_i positive), then it would have to be true that $s_i < \frac{1}{2}$ for some region i, the one with the lower level of G_i. But then $\Phi(s_i) > 0$, so that the second–order conditions are not satisfied for region i at the prospective Nash equilibrium. $\bullet$

Lemma 4 *i* if $\frac{a}{1-a}\frac{g}{1-g} \geq 4$ then there is no symmetric Nash equilibrium

ii if $2 \leq \frac{a}{1-a}\frac{g}{1-g} < 4$ then $G_1 = G_2 = G^E$ is a "local" Nash equilibrium, in that neither region can gain from a small change in its own public investment ; however, the Nash equilibrium is unstable, in that reaction functions have a slope greater than or equal to 1 in absolute value

iii if $\frac{a}{1-a}\frac{g}{1-g} < 2$ then there is a stable local Nash equilibrium ;

Proof. From the implicit function theorem, at any Nash equilibrium, in which $G_i > 0$, the slope of a region's reaction function is defined as

$$\frac{\partial G_i^R}{\partial G_k} = -a\gamma s_k \frac{G_i(s_i(1-a) - 1 - a/2)}{G_k \Phi(s_i)} \tag{38}$$

When $G_1 = G_2$, the slope of the reaction function at a potential equilibrium is

$$-\frac{a\gamma}{4(1-g) - a\gamma}$$

If the expression in the denominator above is positive, then $\Phi(\frac{1}{2}) > 0$, and the prospective symmetric Nash equilibrium does not satisfy the second–order conditions for optimality. If the denominator is negative, but the numerator is larger in absolute value, then reaction functions (in the neighbourhood of the symmetric equilibrium) have a slope greater than 1, so that the equilibrium is "unstable" in the sense that small perturbations will not lead to a new nearby equilibrium.

Proposition 2 When regions' decision makers are xenophobic, and choose public investment levels simultaneously and non–cooperatively, then $G_1 = G_2 = G^E \equiv [\frac{g}{c}]^{1/(1-g)}[\frac{\bar{L}}{2}]^{a/(1-g)}$ will be a Nash equilibrium if and only if $b_X \leq 0$.

Proof. The necessity of the condition $b_X < 0$ has already been shown : if $b_X > 0$ then $\pi^{XE} < \hat{\pi}_i^X$ so that one region would want to deviate to $G_i = 0$ when the other region chose an investment level of G^E.

To demonstrate sufficiency, it must be shown that the second–order conditions for a maximum are satisfied if $b_X \leq 0$; Lemma 2 shows that the only possible best reactions to $G_2 = G^E$ are $G_1 = 0$ and $G_1 = G^E$ if G^E is a local maximum for π_1^X.

If $b_X \leq 0$, then Lemma 1 shows that $b_B < 0$.

So it remains to prove that, $\frac{a}{1-a}\frac{g}{1-g} < 2$ when $b_B < 0$. Since this fraction increases with g and a, it is sufficient to prove that $\frac{a}{1-a}\frac{g}{1-g} < 2$ when $b_B = 0$

If $b_B = 0$, then

$$a = 1 - g(2-g)$$

so that

$$\frac{a}{1-a}\frac{g}{1-g} = \frac{1-g(2-g)}{(1-g)(2-g)}$$

and $\frac{a}{1-a}\frac{g}{1-g} < 2$ at $b_B = 0$ if and only if

$$1 - g(2-g) - 2(1-g)(2-g) < 0 \tag{39}$$

The left side of inequality (39) is an increasing function of g, for $0 < g < 1$, since its derivative with respect to g is $6 - 2g$. At $g = 1$, this left side of inequality (39) equals 0. So (39) must hold for all $0 < g < 1$, implying that $\frac{a}{1-a}\frac{g}{1-g} < 2$ whenever $b_B = 0$. •

Lemma 5 Let $G_1 = G^R(G_2)$ be the value of G_1 which satisfies region 1's first–order condition (15), when region 2's public input is G_2. Then

$$\frac{\partial}{\partial G_2}[\pi_1^X(0, G_2) - \pi_1^X(G^R(G_2), G_2)] > 0$$

(where $\pi_i^X(G_1, G_2)$ denotes the payoff to region 1 when it chooses a public investment level of G_1 and region 2 chooses G_2).

Proof. If $G_1 = G^R(G_2)$, then G_1 satisfies the first–order condition for optimality. This implies that

$$\frac{\partial \pi_1^X(G^R(G_2), G_2)}{\partial G_2} = \gamma G_1^{\gamma}(s_1 \bar{L})^a \frac{a}{s_1}(\frac{1}{2s_1} - 1)(1-a)\frac{s_1 s_2}{G_2}$$

Using the definition of γ, and the fact that $w_1 = w_2$, this equation can be simplified to

$$\frac{\partial \pi_1^X(G^R(G_2), G_2)}{\partial G_2} = a(\frac{1}{2} - s_1)gG_2^{g-1}s_2^a\bar{L}^a$$

Since

$$\frac{\partial \pi_1^X(0, G_2)}{\partial G_2} = \frac{a}{2}gG_2^{g-1}\bar{L}^a$$

the first expression must be smaller than the second, proving the lemma. •

Lemma 6 *(i)* The reaction function $G_2 = G^R(G_1)$ has a positive slope if and only if $s_2 > \frac{2+a}{2+2a} > \frac{3}{4}$.

(iii) The slope of the reaction function $G_2 = G^R(G_1)$ equals 0 at $G_1 = 0$.

(iii) $G^R(G) > G$ if and only if $G < G^E$.

(iv) if $G_1 = G^E$, then $\frac{\partial G^R}{\partial G_1} = -\frac{a\gamma}{4(1-g)-a\gamma}$

Proof. The slope of the reaction function $G_2 = G^R(G_1)$ will be $-\pi_{21}^X/\pi^X 22$, where $\pi^X 2i$ is $\frac{\partial^2 \pi_2^X}{\partial G_2 \partial G_i}$, evaluated at the (unique) $G_2 > 0$ at which $\frac{\partial \pi_2^X}{\partial G_2} = 0$.

Differentiating equation (15),

$$\pi_{22}^X = -(1-g)gG_2^{g-2}(s_2\bar{L})^a(1 + a(\frac{1}{2} - s_2)) + gG_2^{g-1}\bar{L}^a(as_2)^{a-1}(1 - s_2 + a(\frac{1}{2} - s_2))\frac{\partial s_2}{\partial G_2}$$

$$\pi_{21}^X = gG_2^{g-1}\bar{L}^a(as_2)^{a-1}(1 - s_2 + a(\frac{1}{2} - s_2))\frac{\partial s_2}{\partial G_1}$$

Using equations (9) and (10)

$$\pi_{22}^X = [gG_2^{g-2}s_2^{a-1}\bar{L}^a][a\gamma s_1 s_2(1 - s_2 + a(\frac{1}{2} - s_2)) - (1-g)s_2(1 + a(\frac{1}{2} - s_2)] \qquad (40)$$

$$\pi_{21}^X = -\frac{G_2}{G_1}[gG_2^{g-2}s_2^{a-1}\bar{L}^a](a\gamma s_1 s_2)[1 - s_2 + a(\frac{1}{2} - s_2] \qquad (41)$$

If G_2 is an optimal reaction to G_1, then second–order conditions imply that $\pi_{22}^X < 0$. Therefore a necessary and sufficient condition for the reaction function to slope up is that $\pi_{21}^X > 0$. This will be the case if and only if the rightmost expression in square brackets in expression (41) is negative, which will be the case if and only if

$$1 - s_2 < a(\frac{1}{2} - s_2)$$

establishing part (i) of the Lemma.

From equations (40) and (41), the slope of the reaction function can be written as $\frac{G_2}{D}$ where

$$D = G_1 - \frac{G_1}{s_1}\frac{1-g}{a\gamma}\frac{1+a(\frac{1}{2}-s_2)}{1-s_2+a(\frac{1}{2}-s_2)}$$

As $G_1 \to 0$, $s_1/G_1 \to 0$ if $G_2 > 0$, since $s_1 = \frac{G_1^\gamma}{G_1^\gamma+G_2^\gamma}$ and $\gamma > 1$. Therefore, as $G_1 \to 0$, $D \to +\infty$, establishing part (ii) of the lemma.

To prove part (iii) note that there is only one value of G, $G = G^E$ at which $G^R(G) = G$. Therefore, the curve $G_2 = G^R(G_1)$ can cross the 45–degree line at most once. Sine $G^R(0) = G^S > G^E > 0$, the curve starts out above the 45–degree line, and crosses it from above at $G_1 = G_2 = G^E$.

Part (iv) follows from evaluation of (40) and (41) when $G_1 = G_2$ and $s_1 = s_2 = \frac{1}{2}$.

Lemma 7 At $G_1 = G^S, G_2 = 0$, $\pi_1^X > \pi_2^X$ if and only if $b_B < 0$.

Proof. From the definition of G^S, that it be region 1's best response to $G_2 = 0$,

$$g(1-\frac{a}{2})F(G^S,\bar{L}) = cG^S \tag{42}$$

When $G_2 = 0$, the difference between the payoffs to the two regions is the share of the immobile factor, minus the cost of the public infrastructure

$$\pi_1^X - \pi_2^X = (1-a)G_1^g\bar{L}^a - cG_1 \tag{43}$$

Equation (42) implies that this difference is positive if and only if $g(1-\frac{a}{2}) < 1-a$. •

Proposition 5 If $b_B > 0$, then $G_1 < G_2$ when regions move sequentially.

Proof. If $G_1 > G_2$, then region 1's payoff is a decreasing function of region 2's public investment.

So let (G_1, G_2) be any pair of investment levels for which $G_1 > G_2$.

Then $\pi_1^X(G_1, 0) > \pi_1^X(G_1, G_2)$

Since G^S is region 1's best reaction to $G_2 = 0$, then (for any $G_1 > G_2$),

$$\pi_1^X(G^S, 0) = \pi_1^X(G^R(0), 0) \geq \pi_1^X(G_1, 0) \geq \pi_1^X(G_1, G_2)$$

.

If $b_B > 0$, then $\pi_1^X(0, G^S) > \pi_1^X(G^S, 0)$, so that choosing an initial investment level of 0 (and inducing region 2 to choose an investment level of G^S) yields a higher payoff to region 1 than any initial investment level G_1 for which $G_2 = G^R(G_1) \leq G_1$. •

Proposition 6 If $b_B > 0$, then a small increase in G_1 from $G_1 = 0$ must lower xenophobic region 1's payoff, when regions move sequentially.

Proof. The proof follows almost immediately from part (*iv*) of Lemma 6. At $G_1 = 0$, the direct effect of a slight increase in G_1 — holding G_2 constant — must be a decrease in π_1^X, since $\frac{\partial \pi_1^X}{\partial G_1} < 0$ at $G_1 = 0$ if $G_2 > 0$. Increasing G_1 would benefit region 1 only if it induced region 2 to increase its own public investment (since π_1^X increases with G_2 if $G_1 < G_2$). When s_1 is small, it is indeed the case that $G^R(G_1)$ slopes up (as shown in part (*i*) of Lemma 6). But the slope of this reaction function is 0 at $G_1 = 0$, so that the direct damage done by a small increase in G_1 dominates the indirect benefits of the induced increase in investment in the other region. •

Lemma 8 $\pi_1^X(G^*, 0) < \pi_2^X(G^*, 0)$

Proof. Since

$$\pi_1^X(G^*, 0) = F(G^*, \bar{L}) - \frac{1}{2} w^* \bar{L} - cG^*$$

and

$$\pi_2^X(G^*, 0) = \frac{1}{2} \bar{L}$$

then the difference between the payoffs is

$$\pi_1^X - \pi_2^X = F(G^*, 0) - w^* \bar{L} - cG^*$$

The facts that $w^* = F_L(G^*, \bar{L})$, that $F_G(G^*, \bar{L}) = c$ at the optimum, an that $F(G, L)$ has increasing returns, then imply that $\pi_1^X - \pi_2^X$ must be negative. •

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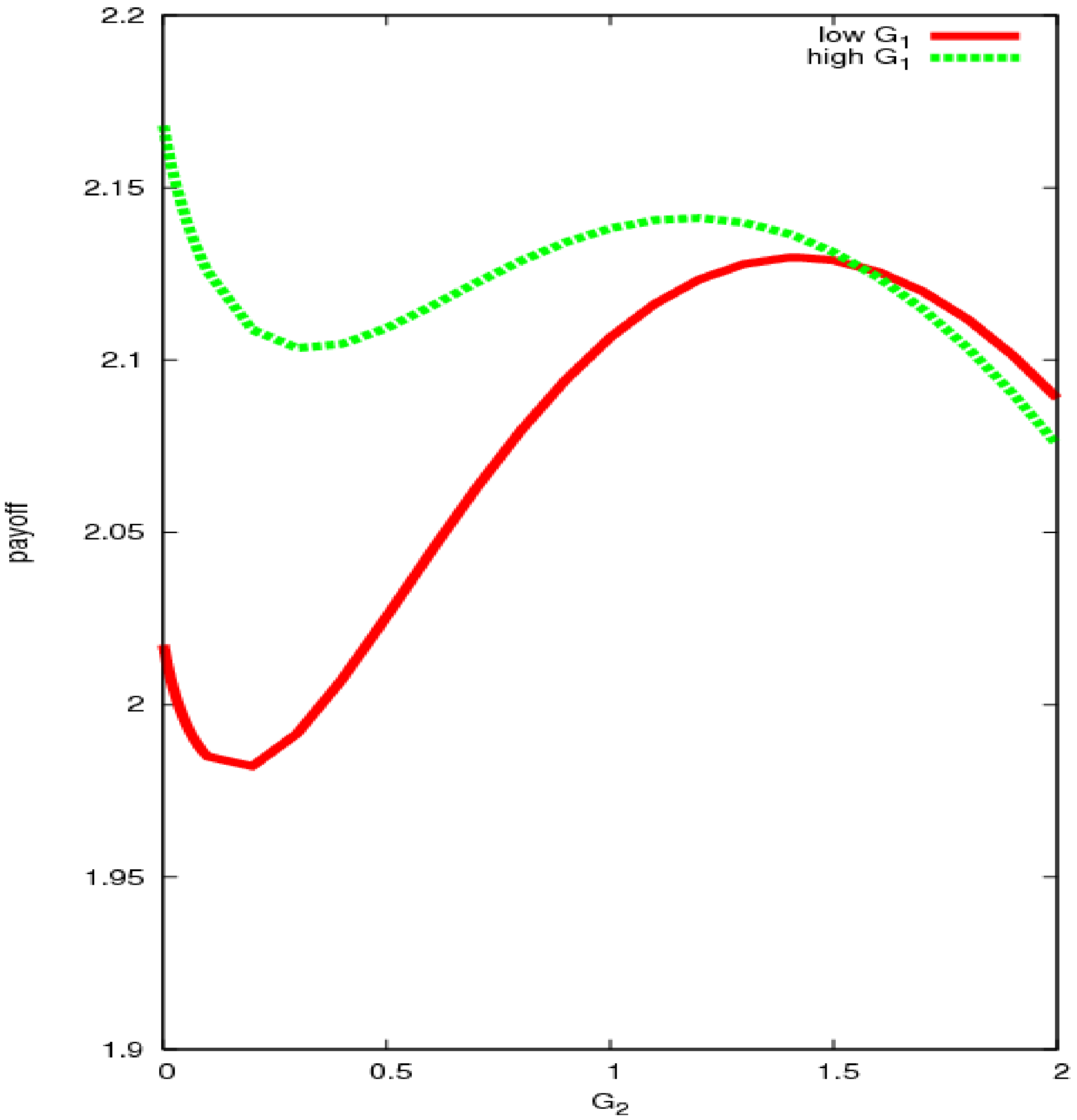

Figure 1 : Region 2's payoff as a function of its level of public investment.

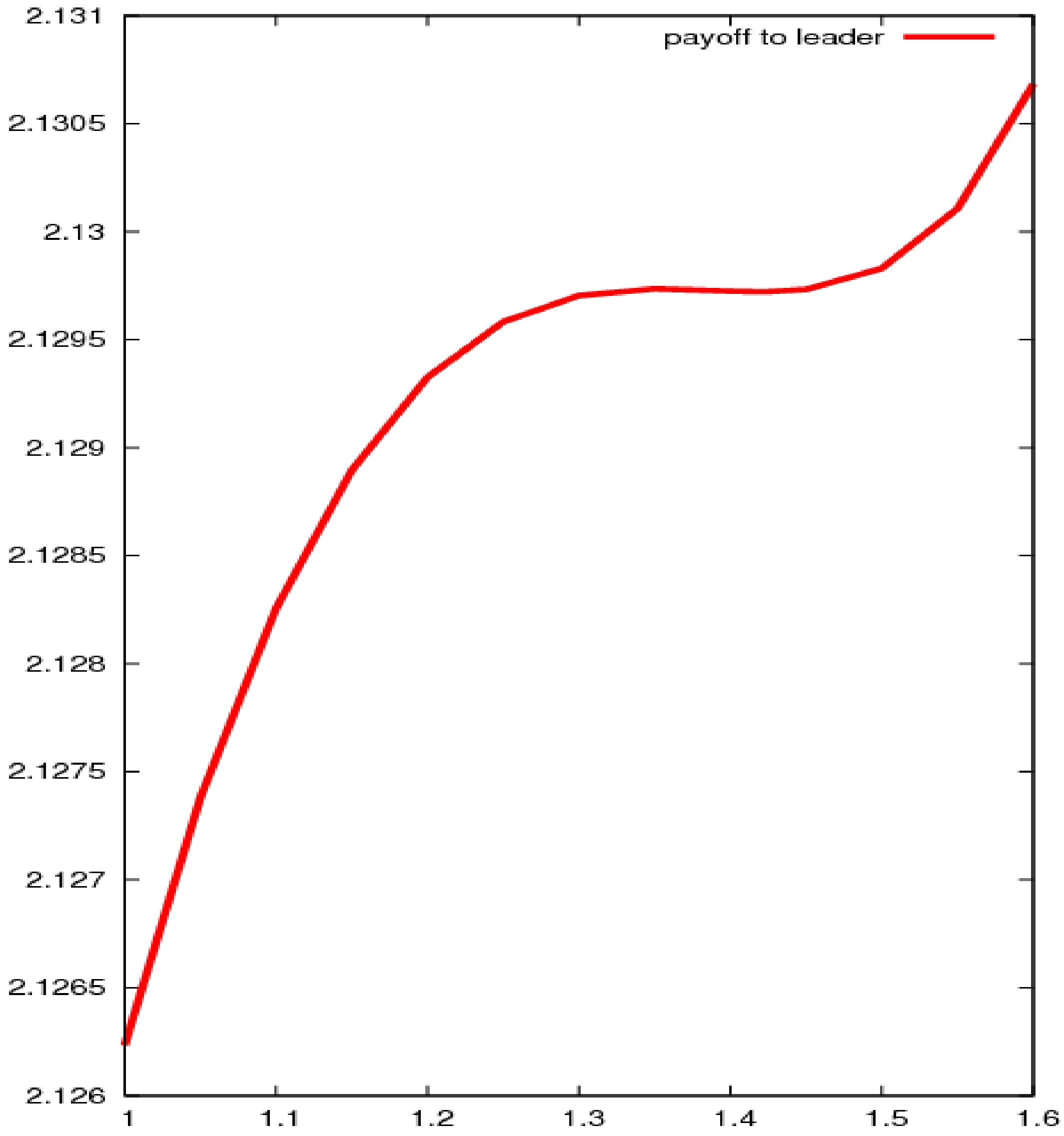

Figure 2 : first mover chooses a higher level of investment than the symmetric Nash equilibrium

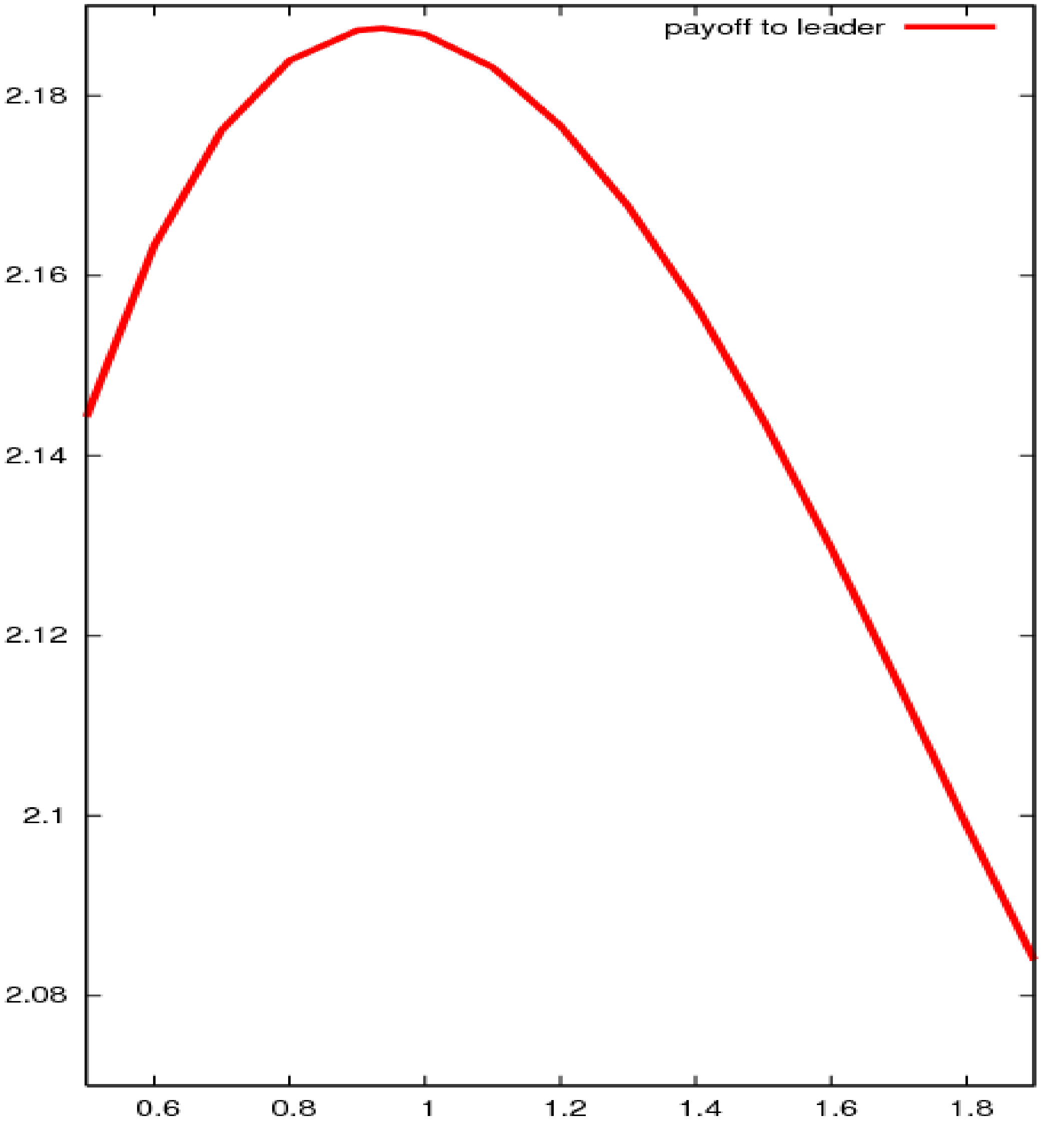

Figure 3 : the same outcome with simultaneous or sequential moves