Appendix 1 : Avoiding Discrete Jumps in Tax Rates

Suppose that country 2 chooses the effective tax rate $\tau^I < f'(e)/(1+\rho)$. As country 1 increases τ_1 above τ^I, it will lose mobile capital. The assumption that the country's payoff function is quasi-concave (when $h_i < k_i$) implies that country 1's payoff decreases as it increases τ_1 further, if τ_1 is already greater than its best response to $\tau_2 = \tau^I$.

However, if τ_1 gets high enough, all mobile capital may move to country 2. This will be the case, at a tax rate τ_1 less than the maximum possible rate $\bar{t}/(1+\rho)$, if the following condition holds

$$f'(2e-h) - \tau^I > f'(h) - \frac{f'(e)}{1+\rho}\,. \tag{A.1}$$

If (A.1) holds, then there is some $\tau^0 \in [\tau^I, f'(e)/(1+\rho)]$ such that $h_1 = k_1$ at $\tau_1 = \tau^0$. In this case further increases in τ_1 above τ^0 have no impact on k_1, as $k_1 = h_1$. Raising τ_1 above τ^0 must then increase the payoff to country 1, as aggregate income of its residents is unchanged, but more income will be diverted to the public sector.

Therefore, country 1's optimal policy, given that the other country has set an effective tax rate of τ^I, is either to choose its interior best response $\tau_1 = \tau^I$, or to choose the maximal possible effective tax rate $f'(e)/(1+\rho)$, and lose all mobile capital. The payoff to the first policy is

$$f(e) + \varepsilon\tau^I(e+\rho h) \tag{A.2}$$

and the payoff from the second policy is

$$f(h) + \varepsilon f'(e)h + \left[f'(2e-h) - \tau^I\right](e-h) \tag{A.3}$$

Thus, given that (A.1) holds, the country will wish to 'deviate' by specializing in immobile capital only if

$$\Delta \equiv \varepsilon[f'(e)h - \tau^I(e+\rho h)] - [f(e) - f(h)] + [f'(2e-h) - \tau^I](e-h) > 0. \tag{A.4}$$

If there were no firms with negative fixed costs of multinational form, so that h equalled 0 for very low values of ρ, then condition (A.4) would have to hold when $h = e$. But our assumption on the cost of multinational form ensures that h is bounded below e, for all values of ρ.

Concavity of the production function $f(\cdot)$ implies that $f'(2e-h) < f'(e)$, so that Δ is bounded above by

$$\varepsilon[f'(e)h - \tau^I(e+\rho h)] - [f(e) - f(h)] + [f'(e) - \tau^I](e-h) > 0.$$

From concavity we also have that $f(e) - f(h) > f'(e)(e-h)$, implying

$$\Delta < \varepsilon f'(e)h - \tau^I[(1+\varepsilon)e - h] \tag{A.5}$$

From equation (12)

$$\tau^I \geq \frac{2\varepsilon}{1+\varepsilon}\frac{f'(e)}{\sigma}\ , \tag{A.6}$$

where σ is the elasticity of capital supply with respect to its net return

$$\sigma \equiv -\frac{f'(e)}{f''(e)e}\ .$$

Equation (A.6) then implies that a sufficient condition for Δ to be negative is that

$$\varepsilon h < \frac{2}{\sigma}\frac{\varepsilon}{(1+\varepsilon)}\ [(1+\varepsilon)e - h].$$

This condition is equivalent to

$$\frac{h}{e} < \frac{2(1+\varepsilon)}{(1+\varepsilon)\sigma + 2}\ . \tag{A.7}$$

Condition (A.7) is a sufficient condition (but not a necessary one) for $\tau_1 = \tau_2 = \tau^I$ to be a Nash equilibrium to the tax–setting stage when $\tau^I < \tau^M$: it implies that a deviation by either country to a maximal statutory tax rate would reduce its payoff. The condition must hold if ε is sufficiently large, or σ sufficiently small.[23]

Condition (A.7) implies fairly weak restrictions on the parameters. For example, Chirinko et al (1999) estimate a value of about 0.25 for the elasticity σ. If this is the case, as long as at least 12 percent of capital were mobile, then condition (A.7) would have to hold for any positive value of for ε.

Chirinko, R., S. Fazzari, and A. Meyer (1999), How responsive is business capital formation to its user cost? An exploration with micro data. *Journal of Public Economics 74*, 53-80

[23]Whenever $\sigma\,(1+\varepsilon)/\varepsilon < 2$, the right side of condition (A.7) must exceed 1. This implies $\tau^I > \tau^M$, so that the tax-setting equilibrium must be in Regime II.

Appendix 2: Extending Proposition 3

Equation (20) defines the first–order condition for a country's choice of tax preferences in the first stage, whatever regime results in stage 3. However, the right side of the equation changes discontinuously at $\rho_1 = \rho_2 = \tilde{\rho}$, the boundary between regimes.

In Regime II, the first term on the right side of (20) does not equal zero, as it does in Regime I. The second term, however, is zero, since the two countries' tax rates are independent of each other when the constraints $\tau_i \leq \tau^M$ bind.

At $\rho_1 = \rho_2 = \tilde{\rho}$, the first term on the right side of (20) will be zero, since there $\tau^I = \tau^M$. Therefore,

$$\left.\frac{\partial u_i}{\partial \rho_i}\right|_{\rho=\tilde{\rho}}^{II} = (1+\varepsilon)\,\tau_i\,h_i\left[\frac{\varepsilon}{1+\varepsilon} - \mu_i\right]. \tag{A.8}$$

If condition (24) holds, then it must be the case that $\mu_i > \varepsilon/(1+\varepsilon)$; otherwise the right side of (23) would be positive. So (24) implies that both $du_i/d\rho_i|^I$ and $du_i/d\rho_i|^{II}$ are negative at $\rho_1 = \rho_2 = \tilde{\rho}$.

On the other hand, if firms are so unresponsive to tax preferences that $\mu_i < \varepsilon/(1+\varepsilon)$ at $\rho_1 = \rho_2 = \tilde{\rho}$, then both $du_i/d\rho_i|^I$ and $du_i/d\rho_i|^{II}$ are positive at $\rho_1 = \rho_2 = \tilde{\rho}$.

Finally, if condition (24) does not hold, but $\mu_i \geq \varepsilon/(1+\varepsilon)$, then $du_i/d\rho_i|^I \geq 0$ and $du_i/d\rho_i|^{II} \leq 0$ at $\rho_1 = \rho_2 = \tilde{\rho}$, so that $\rho_1 = \rho_2 = \tilde{\rho}$ is a Nash equilibrium for the countries' tax preferences.

Summarizing, we get the following extended version of Proposition 3:

Proposition 3* *If $\tilde{\mu}$ denotes the value of $\mu_i (= \mu_j)$ when $\rho_1 = \rho_2 = \tilde{\rho}$ and*

$$\mu^c = 1 - \frac{3+4\varepsilon}{3+8\varepsilon+6\varepsilon^2}$$

then the following holds:

(i) If $\tilde{\mu} > \mu^c$ then there is a symmetric Nash equilibrium in which $\rho_1 = \rho_2 < \tilde{\rho}$ and in which the resulting third–stage tax rates are in Regime I;

(ii) If $\mu^c \geq \tilde{\mu} \geq \varepsilon/(1+\varepsilon)$ then there is a symmetric Nash equilibrium in which $\rho_1 = \rho_2 = \tilde{\rho}$ and in which the resulting third stage tax rates are $\tau^I = \tau^M$;

(iii) If $\varepsilon/(1+\varepsilon) > \tilde{\mu}$ then there is a symmetric Nash equilibrium in which $\rho_1 = \rho_2 > \tilde{\rho}$ and in which the resulting third stage tax rates are in Regime II.

Appendix 3: Proof of Proposition 4

If $\rho_1 = \rho_2$ initially, and if the symmetric third–stage tax–setting equilibrium is in Regime I, then the equilibrium values of τ_1, τ_2, h_1 and h_2 can be defined as the solution to the system of four equations

$$(e - k_i)\frac{\partial r}{\partial \tau_i} + \varepsilon(k_i + \rho_i h_i) + (1+\varepsilon)\,\tau_i\,\frac{\partial k_i}{\partial \tau_i} = 0 \quad \forall\, i \in \{1,2\}, \tag{A.9}$$

$$h_i - \int_{\rho_i \tau_i}^{\infty} g(c)dc = 0 \quad \forall\, i \in \{1,2\}. \tag{A.10}$$

A symmetric equilibrium is further characterized by

$$\frac{\partial k_i}{\partial \tau_i} = \frac{1}{2f''(e)}, \qquad \frac{\partial r}{\partial \tau_i} = -\frac{1}{2}, \qquad \frac{\partial^2 k_i}{\partial {\tau_i}^2} = 0. \tag{A.11}$$

Equation (A.9) defines the reaction curve for a country in the final, tax-setting stage. From equation set (A.11), the slope of a reaction curve, in a symmetric equilibrium is

$$\frac{\partial \tau_i}{\partial \tau_j} = \frac{1+2\varepsilon}{3+4\varepsilon} \quad i \neq j\ . \tag{A.12}$$

Also using the results (A.11), the differential of the equation system (A.9)–(A.10) can be written

$$\begin{pmatrix} \frac{3+4\varepsilon}{4f''(e)} & -\frac{1+2\varepsilon}{4f''(e)} & \varepsilon\rho & 0 \\ -\frac{1+2\varepsilon}{4f''(e)} & \frac{3+4\varepsilon}{4f''(e)} & 0 & \varepsilon\rho \\ \rho g(\rho\tau) & 0 & 1 & 0 \\ 0 & \rho g(\rho\tau) & 0 & 1 \end{pmatrix} \begin{pmatrix} d\tau_1 \\ d\tau_2 \\ dh_1 \\ dh_2 \end{pmatrix} = \begin{pmatrix} -\varepsilon h \\ 0 \\ -\tau g(\rho\tau) \\ 0 \end{pmatrix} d\rho_1 \tag{A.13}$$

The determinant of the matrix on the left side of equation (A.13) is

$$\Delta = A + B$$

where

$$A \equiv \frac{(3+4\varepsilon)^2 - (1+2\varepsilon)^2}{16[f''(e)]^2} - \varepsilon\rho^2 g(\rho\tau)\frac{3+4\varepsilon}{4f''(e)} > 0, \tag{A.14}$$

$$B \equiv \varepsilon^2\rho^4[g(\rho\tau)]^2 - \varepsilon\rho^2 g(\rho\tau)\frac{3+4\varepsilon}{4f''(e)} > 0. \tag{A.15}$$

Cramer's Rule then shows the effects on the subsequent stages of a unilateral change in one country's tax preferences

$$\frac{dh_i}{d\rho_i} = -\frac{\tau g(\rho\tau)A + (h/\rho)B}{A+B}\ . \tag{A.16}$$

From the definition of the elasticity of firm structure with respect to the tax advantages of MNE form [eq. (A.27) in the main text] and (A.10)

$$\eta \equiv -\frac{dh}{d(\rho\tau)}\frac{\rho\tau}{h} = g(\rho\tau)\,\frac{\rho\tau}{h}\ ,$$

so that equation (A.16) becomes

$$\frac{dh_i}{d\rho_i} = -\frac{h}{\rho}\left[1+(\eta-1)\frac{A}{A+B}\right] \quad\Longleftrightarrow\quad 1-\mu_i = \frac{A}{A+B}\,(1-\eta), \tag{A.17}$$

where the definition of μ_i in the main text [eq. (22)] has been used. It follows that at a symmetric equilibrium in Regime I:

Lemma A.1 *If $\eta < 1$ ($\eta > 1$) then $\eta < \mu < 1$ ($\eta > \mu > 1$).*

Further, equation (A.13) implies that

$$\frac{dh_j}{d\rho_i} = (1-\eta)\,\frac{\varepsilon\, h\, \rho\, g(\rho\tau)}{(A+B)}\,\frac{(1+2\varepsilon)}{4f''(e)} \quad j \neq i, \tag{A.18}$$

so that an increase in one country's tax preferences will decrease the number of immobile firms in the other country if $\eta < 1$.

For the response of a country's effective tax rate with respect to its own tax preference parameter, Cramer's Rule gives

$$\frac{d\tau_i}{d\rho_i} = (1-\eta)\,\frac{\varepsilon h}{A+B}\left[\varepsilon\rho^2 g(\tau\rho) - \frac{3+4\varepsilon}{4f''(e)}\right], \tag{A.19}$$

so that $d\tau_i/d\rho_i > 0$ if and only if $\eta < 1$. Moreover, if $\eta < 1$, then $d\tau_j/d\rho_i > 0$ also holds from the fact that reaction curves slope up near a symmetric equilibrium.

Lemma A.2 *If $0 \le \varepsilon \le 1$, then a coordinated increase in the tax preference parameter ρ must increase the payoff to each country, starting from a non–cooperative equilibrium which implies an outcome in Regime I.*

Proof: It is necessary to show that an increase in ρ_i must raise country j's payoff, starting from a symmetric Nash equilibrium (for which the stage 3 tax rates are in Regime I).

The effect of ρ_i on u_j was defined by equation (25a) of the text, repeated here

$$\left.\frac{\partial u_j}{\partial \rho_i}\right|^{I} = (1+\varepsilon)\,\tau_j\left(\frac{\partial k_j}{\partial \tau_i}\left.\frac{d\tau_i}{d\rho_i}\right|^{I} + \rho_j\frac{dh_j}{d\rho_i}\right) \quad \forall\, i \neq j$$

Equation (11) of the text implies that

$$\varepsilon(e+\rho h) = -(1+\varepsilon)\tau\frac{\partial k_i}{\partial \tau_i} \quad \Longleftrightarrow \quad \varepsilon(e+\rho h) = (1+\varepsilon)\tau\frac{\partial k_j}{\partial \tau_i} \quad j \neq i$$

At a symmetric non-cooperative equilibrium, equation (20) can be written

$$\left.\frac{\partial u_i}{\partial \rho_i}\right|^I = \varepsilon(e+\rho\, h)\ \frac{\partial \tau_j}{\partial \tau_i}\ \frac{d\tau_i}{d\rho_i} + \tau_i \left[\varepsilon\, h_i + (1+\varepsilon)\ \rho_i \frac{dh_i}{d\rho_i}\right] = 0,$$

if the third–stage equilibrium is in Regime I. From the definition of μ, and equation (A.12), this becomes

$$\varepsilon(e+\rho h)\frac{d\tau_i}{d\rho_i} = -\tau h\left[\varepsilon - (1+\varepsilon\mu)\right]\ \frac{3+4\varepsilon}{1+2\varepsilon}\ . \tag{A.20}$$

so that

$$\left.\frac{\partial u_j}{\partial \rho_i}\right|^I = \tau h\frac{3+4\varepsilon}{1+2\varepsilon}[1-\varepsilon(1-\mu)] + \rho_j(1+\varepsilon)\tau\frac{dh_j}{d\rho_i} \quad \forall\, i \neq j$$

Using (A.18), this becomes

$$\left.\frac{\partial u_j}{\partial \rho_i}\right|^I = \tau h\left[\frac{3+4\varepsilon}{1+2\varepsilon}(1-\varepsilon(1-\mu)) - \frac{(1+\varepsilon)\rho^2 g(\rho\tau)\varepsilon(1-\eta)(1+2\varepsilon)}{-4f''(e)\ (A+B)}\right]\ . \tag{A.21}$$

¿From the definition (A.14)

$$\varepsilon\ \rho^2 g(\rho\tau)\ \frac{1+2\varepsilon}{-4f''(e)} < \frac{A}{2}$$

so that

$$\left.\frac{\partial u_j}{\partial \rho_i}\right|^I > \tau h\left[\frac{3+4\varepsilon}{1+2\varepsilon}(1-\varepsilon(1-\mu))\ - \frac{1}{2}(1+\varepsilon)\frac{A}{A+B}\right]\ .$$

This implies that $\partial u_j/\partial \rho_i|^I > 0$ when

$$(3+4\varepsilon)\left[1-\varepsilon(1-\mu)\right] - \frac{1}{2}(1+\varepsilon)(1+2\varepsilon)\frac{A}{A+B} > 0. \tag{A.22}$$

Since $\mu \geq \varepsilon/(1+\varepsilon)$ if the non-cooperative equilibrium leads to an outcome in Regime I, the left side of equation (A.22) is greater than or equal to

$$(3+4\varepsilon)(1-\varepsilon) + \varepsilon^2\frac{3+4\varepsilon}{1+\varepsilon} - \frac{1}{2}(1+\varepsilon)(1+2\varepsilon)\frac{A}{A+B} \tag{A.23}$$

Since $3+4\varepsilon > 3(1+\varepsilon)$, and since A and B are positive, so that $A/(A+B) < 1$, expression (A.23) is strictly greater than

$$(3+4\varepsilon)(1-\varepsilon) + 3\varepsilon^2 - \frac{1}{2}(1+\varepsilon)(1+2\varepsilon) \tag{A.24}$$

which in turn can be simplified to

$$\frac{1}{2}\left[5 - \varepsilon - 4\varepsilon^2\right] \tag{A.25}$$

Expression (A.25) must be non–negative for all $0 \leq \varepsilon \leq 1$. Therefore, (A.23) must be strictly positive whenever $0 \leq \varepsilon \leq 1$, so that inequality (A.22) must hold whenever there is a Nash equilibrium leading to an outcome in Regime I when $\varepsilon \leq 1$. •

When $\varepsilon > 1$, a similar result can be shown.

Lemma A.3 *If $\varepsilon \geq 1$, then a coordinated increase in the tax preference parameter ρ must increase the payoff to each country, starting from a non–cooperative equilibrium which implies an outcome in Regime I.*

Proof. From equations (25a), (A.18), and (A.19), we have

$$\left.\frac{\partial u_j}{\partial \rho_i}\right|^I = \frac{(1-\eta)\,\varepsilon^2 h\,(e+\rho h)}{A+B}\left[\varepsilon\rho^2 g(\rho\tau) + \frac{3+4\varepsilon}{[-4f''(e)]}\right] - \frac{(1-\eta)\,(1+\varepsilon)\,\rho\,\eta\,\varepsilon\,h^2(1+2\varepsilon)}{(A+B)\,[-4f''(e)]}$$

Since $\eta < 1$ at any non–cooperative equilibrium leading to an outcome in Regime I, and since A and B are both positive, this effect will be positive iff

$$\varepsilon(e+\rho h)\left[\varepsilon\rho^2 g(\rho\tau) + \frac{3+4\varepsilon}{[-4f''(e)]}\right] - \frac{\eta\,\rho\,h\,(1+\varepsilon)\,(1+2\varepsilon)}{[-4f''(e)]} > 0.$$

But since $e + \rho h > \rho h$, and $3 + 4\varepsilon > 2(1 + 2\varepsilon)$, this will be positive whenever

$$\eta \leq \frac{2\varepsilon}{1+\varepsilon}\ . \tag{A.26}$$

At the non–cooperative equilibrium $\eta < 1$. The right side of inequality (A.26) equals 1 when $\varepsilon = 1$, and is an increasing function of ε. •

Lemma A.2 and Lemma A.3 together complete the proof of Proposition 4, if the final-stage equilibrium is in Regime I. □

Appendix 4: Optimal coordinated discrimination policies

The analysis of coordinated discrimination policies in the main text has been confined to small coordinated changes in discrimination policies, starting from a non-cooperative equilibrium. This appendix asks instead what is the degree of tax discrimination that maximizes joint welfare. Within Regime II, equation (25b) shows that welfare must monotonously decline with the degree of tax preferences. In Regime I our analysis has shown that the net effect in (25a) is positive for a small increase in ρ above the non–cooperative level. It is not clear, however, that ρ should be increased all the way to the boundary between the two regimes, given by $\tilde{\rho}$. The reason for this ambiguity is that the benefits of decreased tax competition may be offset by the increases in total fixed costs incurred by firms.

Whether discrimination should be increased or decreased within Regime I is determined by the elasticity of firm structure with respect to the *coordinated* tax advantage $\rho\tau$ of multinational form. This elasticity is defined by

$$\eta \equiv -\frac{dh}{d\rho\tau}\frac{\rho\tau}{h} = \frac{g(\rho\tau)\rho\tau}{\int_{\rho\tau}^{\infty} g(c)dc} > 0\ , \qquad \text{(A.27)}$$

where the second step uses (18). We can state the following condition for η which ensures that coordinated increases ρ are welfare-enhancing throughout Regime I:

Proposition A.1 *The optimal coordinated discrimination policy cannot exceed $\tilde{\rho}$. If $\eta \le \varepsilon/(1+\varepsilon)$, then the optimal coordinated discrimination policy equals $\tilde{\rho}$, and maximizes the effective tax rate set in the last stage of the game.*

Proof: Equation (25b) establishes that a reduction in ρ_i must increase u_j throughout Regime II. In Regime I, consider the effect of a coordinated change in ρ. Equations (17) and (12) imply that

$$h - \int_{\rho C(e+\rho h)}^{\infty} g(c)dc = 0 \qquad \text{(A.28)}$$

where $C \equiv [\varepsilon/(1+\varepsilon)][-2f''(e)] > 0$. Differentiation of (A.28) yields

$$\left.\frac{dh}{d\rho}\right|^{c} = -\frac{h(e+2\rho h)\,\eta}{\rho[e+(1+\eta)\rho h]} \qquad \text{(A.29)}$$

where the superscript c is used to denote a simultaneous (coordinated) policy change in both countries. Also, since $\tau = C(e + \rho h)$,

$$\left.\frac{d\tau}{d\rho}\right|^{c} = Ch\left(1 + \frac{\rho}{h}\frac{dh}{d\rho}\right) . \tag{A.30}$$

In a symmetric equilibrium, where each country employs a level of capital $k_i = e$, the payoff u to each country's government can thus be written as

$$u = f(e) - \int_{-\infty}^{\rho\tau} cg(c)dc + \varepsilon\tau\,(e + \rho h) = f(e) - \int_{-\infty}^{\rho\tau} cg(c)dc + \varepsilon\,\frac{\tau^2}{C} . \tag{A.31}$$

Note that a coordinated increase in ρ must increase the number of mobile firms in each country [from equation (A.29)]. Thus a necessary condition for this increase to be welfare–improving is that total tax revenue rises. If $\eta \geq 1$, then equation (A.30) shows that τ, and hence tax revenue falls. Therefore $\eta < 1$ is a necessary condition for an increase in ρ to increase utility in each country.

But using (A.30) and (A.31), $du/d\rho$ can be shown to be proportional to

$$2\varepsilon(e + \rho h) - \eta\left[2\varepsilon\,(e + \rho h) + e + 2\rho h\right]$$

so that, if $du/d\rho = 0$, then

$$\frac{\varepsilon}{1 + \varepsilon} < \eta < \frac{2\varepsilon}{1 + 2\varepsilon}$$

holds in Regime I. From this follows that u will be monotonously increasing in ρ throughout Regime I when $\eta < \varepsilon/(1 + \varepsilon)$. □

Hence, if η is sufficiently low, then the gain in tax revenues resulting from a joint increase in ρ dominates the induced increase in firms' fixed costs throughout Regime I. In this case countries will jointly choose the discrimination policy that induces each one of them to levy the highest possible level of τ in the non-cooperative final stage of the game. But this level is reached just at the boundary between the two regimes, as τ is rising in ρ in Regime I, but falling in ρ in Regime II.

Appendix 5: Differences in country size

Assumptions

The two countries may differ in population, but are otherwise identical. In particular, each country has the same excess burden ε of the tax system, and the same per capita endowment of capital e. Let s_i denote the share of the total (immobile) population which resides in country i, so that $s_1 + s_2 = 1$. We adopt the convention that country 1 has the larger population, $s_1 \geq s_2$.

In order to obtain analytic results, two strong restrictions will be imposed upon the forms for the production function and the density function for firms' fixed costs of multinational form.

(i) the production function is: $f(k_i) \equiv ak_i - (b/2)k_i^2 \quad \forall \quad i = 1, 2, \quad a > 0, \quad b > 0.$

(ii) the distribution of fixed costs is: $g(c) = \beta > 0, \quad 0 \leq c \leq A.$

Let α denote the fraction of firms which choose multinational form, even when there are no tax preferences

$$\alpha \equiv \int_{\underline{\mathrm{C}}}^{0} g(c)dc$$

Then it must be the case that

$$\alpha + \beta A < 1$$

so that some firms choose not to become multinationals when the tax advantage of multinational form is sufficiently small.

It will be assumed that

$$\frac{as_1s_2}{be} > \frac{2s_1s_2 + \varepsilon(5s_1s_2 - 1) - \varepsilon^2(s_1 - s_2)^2}{3\varepsilon^2 + 5\varepsilon + 2} \; . \tag{A.32}$$

Restriction (A.32) ensures a positive net return to capital in the Nash equilibrium to the tax–setting game played by countries in the third stage, if each country does not have any tax preferences ($\rho_1 = \rho_2 = 0$). Since all the results obtained in this appendix will hold only in a neighbourhood of $\rho_1 = \rho_2 = 0$, this restriction is needed to ensure that the third–stage outcome is in Regime I, if tax preferences in each country are small enough. Note that restriction (A.32) requires that, for given excess burden parameter ε, the ratio a/be be sufficiently large, if size asymmetries are large ($s_2 \to 0$).

For a Nash equilibrium in Regime I, it is also required that neither country prefers to set its own tax rate so high that all mobile firms move to the other country, leaving the deviating country free to confiscate all the return to domestic immobile capital and use it for public expenditure. Such deviations to a high–tax, no–mobile–capital policy are not possible (i.e., tax increases in country i drive capital's net return down to zero before they drive out all mobile capital), if the following condition is fulfilled

$$be\varepsilon[s_i(1+\varepsilon)+2s_i(1-s_i)+\varepsilon] > (1-s_i)(3\varepsilon^2+5\varepsilon+2)[as_i-be(1-\alpha(1-s_i))]. \quad \text{(A.33)}$$

Even if condition (A.33) does not hold, countries still might not want to deviate to this high–tax policy. We do not provide a further closed–form condition for these deviations not to be beneficial. However, in every one of the calculated examples provided below, it was confirmed that neither country did want to deviate, and that the outcomes presented as (stage 3) tax–setting Nash equilibria in Regime I are indeed global best responses for each country.

That is, even though each country places a premium on public sector expenditure, and even though (in the examples) most of the capital is immobile, countries will not choose simply to confiscate the rents to the immobile capital; they each have an incentive to try and attract some of the mobile capital.

The Effect of One Country's Tax Rate on the Other Country

Proposition A.4 below uses the following characterization of the effect of one country's tax rate on the other country's well–being:

Lemma A.4 *Holding constant tax preferences, and the number of multinational firms, the effect on one country's effective tax rate on the other country's payoff is*

$$\frac{\partial u_i}{\partial \tau_j} = (1+\varepsilon)k_i - e \quad \text{(A.34)}$$

Proof: In general, even when the production function is not quadratic, the first–order condition for optimality of country i's tax rate can be written

$$(1+\varepsilon)\tau_i\frac{\partial k_i}{\partial \tau_i} + (e-k_i)\frac{\partial r}{\partial \tau_i} + \varepsilon k_i = 0\,. \quad \text{(A.35)}$$

The effect of country j's tax rate on utility in country i is

$$\frac{\partial u_i}{\partial \tau_j} = (1+\varepsilon)\tau_i \frac{\partial k_i}{\partial \tau_j} + (e - k_i)\frac{\partial r}{\partial \tau_j} \ . \tag{A.36}$$

With fixed aggregate endowments, it must be the case that

$$\frac{\partial k_i}{\partial \tau_j} = -\frac{\partial k_i}{\partial \tau_j} \tag{A.37}$$

and that

$$\frac{\partial r}{\partial \tau_i} + \frac{\partial r}{\partial \tau_j} = -1. \tag{A.38}$$

Substitution of (A.37), (A.38) and (A.35) into (A.36) implies (A.34). •

Deriving closed-form solutions for τ_i and h_i

The quadratic production function implies that

$$f'(k_i) = a - bk_i$$

so that the equation defining the net return to capital in each region is

$$r = a - bk_1 - \tau_1 = a - bk_2 - \tau_2 \ . \tag{A.39}$$

When regions differ in size, full employment of the total endowment of capital implies

$$s_1 k_1 + s_2 k_2 = e \equiv \bar{e} \tag{A.40}$$

Equations (A.39) and (A.40) imply per-capita capital stocks

$$k_1 = e - \frac{s_2}{b}(\tau_1 - \tau_2), \qquad k_2 = e - \frac{s_1}{b}(\tau_2 - \tau_1), \tag{A.41}$$

which in turn determine the net return to mobile capital as

$$r = \bar{r} - s_1\tau_1 - s_2\tau_2, \qquad \bar{r} \equiv a - be. \tag{A.42}$$

From equations (A.41)–(A.42),

$$\frac{\partial k_i}{\partial \tau_i} = -\frac{1-s_i}{b}, \qquad \frac{\partial k_j}{\partial \tau_i} = \frac{s_i}{b} \ \forall \ j \neq i, \qquad \frac{\partial r}{\partial \tau_i} = -s_i \ . \tag{A.43}$$

The first–order condition for country i's choice of its own effective tax rate τ_i in the final stage of the game is therefore

$$\varepsilon(k_i + \rho_i h_i) + (1+\varepsilon)\tau_i \frac{\partial k_i}{\partial \tau_i} + (e - k_i)\frac{\partial r}{\partial \tau_i} = 0. \tag{A.44}$$

Substituting into (A.44) from (A.41) and (A.43), country 1's optimal choice of effective tax rate obeys

$$\varepsilon\left[e-\frac{s_2}{b}(\tau_1-\tau_2)+\rho_1 h_1\right]-\tau_1(1+\varepsilon)\frac{s_2}{b}-\frac{s_2}{b}(\tau_1-\tau_2)s_1=0$$

which in turn can be written

$$b\varepsilon+b\varepsilon\rho_1 h_1-s_2[1+s_1+2\varepsilon]\tau_1+s_2[s_1+\varepsilon]\tau_2=0. \tag{A.45}$$

The analogous first–order condition for country 2 is

$$b\varepsilon+b\varepsilon\rho_2 h_2-s_1[1+s_2+2\varepsilon]\tau_2+s_1[s_2+\varepsilon]\tau_1=0 \tag{A.46}$$

The effective tax rates in the two countries (if the outcome is in Regime I) are defined by the equation

$$A\begin{pmatrix}\tau_1\\ \tau_2\end{pmatrix}=\begin{pmatrix}b\varepsilon e\\ b\varepsilon e\end{pmatrix}+\begin{pmatrix}b\varepsilon_1\\ 0\end{pmatrix}\rho_1 h_1+\begin{pmatrix}0\\ b\varepsilon_2\end{pmatrix}\rho_2 h_2 \tag{A.47}$$

where the elements of the matrix A are defined by

$$A_{ii}=s_j(1+s_i+2\varepsilon),\qquad A_{ij}=-s_j(s_i+\varepsilon)\quad j\neq i$$

The determinant Δ_A of the matrix A is

$$\Delta_A=s_1s_2\left(3\varepsilon^2+5\varepsilon+2\right)>0.$$

Solving the linear equation system (A.47) implies that

$$\tau_i=B_{ii}\rho_i h_i+B_{ij}\rho_j h_j+[B_{ii}+B_{ij}]e\quad j\neq i \tag{A.48}$$

where

$$B_{ii}=\frac{b}{\Delta_A}(s_i(1+s_j+2\varepsilon))\varepsilon>0,$$

$$B_{ij}=\frac{b}{\Delta_A}(s_j(s_i+\varepsilon))\varepsilon>0,\quad j\neq i.$$

Equation (A.48) gives a closed–form expression for the final–stage effective tax rates, in terms of the tax preferences chosen in the initial stage, and the number of immobile firms in each country chosen in the second stage.

Turning to the second stage, from the assumption on the density function for fixed costs follows

$$h_i = [\alpha - \beta\rho_i\tau_i]e \quad i = 1, 2 \tag{A.49}$$

which, from (A.48), gives

$$h_i = \alpha e_i - \beta[B_{ii}\rho_i h_i + B_{ij}\rho_j h_j + (B_{ii} + B_{ij})]\, \rho_i e_i. \tag{A.50}$$

Hence

$$D \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \alpha e_1 - \beta(B_{11} + B_{12})\rho_1 e^2 \\ \alpha e_2 - \beta(B_{21} + B_{22})\rho_2 e^2 \end{pmatrix} \tag{A.51}$$

where the elements of the matrix D are defined as

$$D_{ii} = 1 + \beta B_{ii} e \rho_i^2,$$

$$D_{ij} = \beta B_{ij}\rho_i\rho_j e, \quad j \neq i.$$

Equation (A.51) provides closed–form expressions for the number of immobile firms in each country, as a function of the countries' tax preferences:

$$h_1 = \frac{e}{\Delta_D}[(\alpha - \beta B_{11}\rho_1 e - \beta B_{12}\rho_1 e)D_{22} - (\alpha - \beta B_{21}\rho_2 e - \beta B_{22}\rho_2 e)D_{12}] \tag{A.52}$$

$$h_2 = \frac{e}{\Delta_D}[(\alpha - \beta B_{21}\rho_2 e - \beta B_{22}\rho_2 e)D_{11} - (\alpha - \beta B_{11}\rho_1 e - \beta B_{12}\rho_1 e)D_{21}] \tag{A.53}$$

where

$$\Delta_D = 1 + \beta(B_{11}\rho_1^2 + B_{22}\rho_2^2)e + \beta^2\rho_1^2\rho_2^2 e^2(B_{11}B_{22} - B_{12}B_{21}).$$

Local results when tax preferences are zero initially

Suppose that we start from an initial situation in which one of the countries, country i, has no preferences for multinationals. The other country, j, has non–negative preferences. If $\rho_i = 0$, then

$$D_{ii} = 1;$$

$$D_{ij} = D_{ji} = 0, \quad j \neq i;$$

$$D_{jj} = 1 + B_{jj}(\rho_j)^2 e, \quad j \neq i;$$

$$\Delta_D = 1 + B_{jj}(\rho_j)^2 e, \quad j \neq i.$$

Further,

$$\frac{\partial D_{ii}}{\partial \rho_i} = \frac{\partial D_{jj}}{\partial \rho_i} = \frac{\partial \Delta_D}{\partial \rho_i} = 0,$$

$$\frac{\partial D_{ij}}{\partial \rho_i} = \beta B_{ij}\rho_j e \geq 0, \qquad \frac{\partial D_{ji}}{\partial \rho_i} = \beta B_{ji}\rho_j e \geq 0.$$

From equations (A.52) and (A.53) this implies

$$\frac{dh_j}{d\rho_i} = \frac{\alpha e}{\Delta_D}\beta B_{ji}\rho_j e \geq 0, \quad j \neq i \tag{A.54}$$

It then can be shown that

Proposition A.2 *Under the assumptions of this appendix, starting from $\rho_i = 0$, a small increase in country i's tax preference ρ_i must increase its own effective tax rate τ_i in the subsequent tax–setting equilibrium, as well as the effective tax rate τ_j in the other country.*

Proof: Equation (A.48) implies that, when $\rho_i = 0$,

$$\frac{d\tau_i}{d\rho_i} = B_{ii}h_i + B_{ij}\rho_j\frac{dh_j}{d\rho_i} \tag{A.55}$$

and that

$$\frac{d\tau_j}{d\rho_i} = B_{jj}h_j + B_{ji}\rho_j\frac{dh_j}{d\rho_i}, \quad j \neq i \tag{A.56}$$

From equation (A.54), the assumption that the number of immobile firms in each region is strictly positive, and the fact that all the B_{mn}'s are positive, expressions (A.55) and (A.56) both must be strictly positive. □

Lemma A.5 *If $\rho_i = 0$, then in the third–stage tax–setting equilibrium*

$$(1+\varepsilon)k_i > e \tag{A.57}$$

Proof: Suppose first that **both** ρ_1 and ρ_2 equal 0. Then $h_1 = h_2 = \alpha e$, and equation (A.48) implies that the third–stage effective tax rates (τ_1^0, τ_2^0) in the two countries are

$$\tau_1^0 = \frac{be}{s_1 s_2}\frac{\varepsilon}{(3\varepsilon^2 + 5\varepsilon + 2)}\,\left[s_1(1+\varepsilon) + 2s_1 s_2 + \varepsilon\right], \tag{A.58}$$

$$\tau_2^0 = \frac{be}{s_1 s_2}\frac{\varepsilon}{(3\varepsilon^2 + 5\varepsilon + 2)}\,\left[s_2(1+\varepsilon) + 2s_1 s_2 + \varepsilon\right]. \tag{A.59}$$

From the definition (A.42) of the net return r, and equations (A.58) and (A.59), condition (A.32) implies that τ_1 and τ_2 are sufficiently small at $\rho_1 = \rho_2 = 0$ that the resulting net return to investment r must be strictly positive.

Equations (A.58) and (A.59) imply that

$$\tau_1^0 - \tau_2^0 = \frac{be}{s_1 s_2} \frac{\varepsilon(1+\varepsilon)}{(3\varepsilon^2 + 5\varepsilon + 2)} (s_1 - s_2). \tag{A.60}$$

Moreover, from equations (A.41), in Nash equilibrium

$$k_1^0 = \frac{2(1+\varepsilon)^2 e}{3\varepsilon^2 + 5\varepsilon + 2} + \frac{s_2}{s_1} \frac{\varepsilon(1+\varepsilon) e}{(3\varepsilon^2 + 5\varepsilon + 2)} \tag{A.61}$$

$$k_2^0 = \frac{2(1+\varepsilon)^2 e}{3\varepsilon^2 + 5\varepsilon + 2} + \frac{s_1}{s_2} \frac{\varepsilon(1+\varepsilon) e}{(3\varepsilon^2 + 5\varepsilon + 2)} \tag{A.62}$$

Equations (A.61) and (A.62) imply that

$$k_i^0 > \frac{2(1+\varepsilon)^2 e}{3\varepsilon^2 + 5\varepsilon + 2}$$

Therefore, condition (A.57) will hold if

$$\frac{2(1+\varepsilon)^3}{3\varepsilon^2 + 5\varepsilon + 2} > 1 \tag{A.63}$$

for all $\varepsilon > 0$.

But condition (A.63) is equivalent to

$$\Phi(\varepsilon) > 0 \quad \text{for} \quad \text{all} \quad \varepsilon > 0 \tag{A.64}$$

where

$$\Phi(\varepsilon) \equiv 2(1+\varepsilon)^3 - (3\varepsilon^2 + 5\varepsilon + 2)$$

Since $\Phi(0) = 0$, and

$$\Phi'(\varepsilon) = 1 + 6\varepsilon + 6\varepsilon^2$$

condition (A.64) must hold. This proves the lemma for the case that $\rho_1 = \rho_2 = 0$.

Now assume that $\rho_i = 0$ and that $\rho_j > 0 \quad (j \neq i)$. When $\rho_i = 0$, equation (A.48) implies that

$$\tau_j - \tau_i = \tau_j^0 - \tau_i^0 + (B_{jj} - B_{ij})\rho_j h_j$$

Since $B_{jj} > B_{ij}$, $\tau_j - \tau_i$ must be strictly larger than $\tau_j^0 - \tau_i^0$ when $\rho_i = 0$ and $\rho_j > 0$. Since k_i is an increasing function of $\tau_j - \tau_i$, therefore k_i must be strictly larger than k_i^0, completing the proof of the lemma. •

The proof of the lemma incorporates the following result, which follows directly from equation (A.60):

Proposition A.3 *If $\rho_1 = \rho_2 = 0$, then the larger country (country 1 by convention) levies the higher effective tax rate in the third-stage tax-setting equilibrium.*

We can now move on to:

Proposition A.4 *Starting from a situation in which $\rho_i = 0$, a slight increase in ρ_i must benefit country j.*

Proof: The payoff to country j is $f(k_j) + r(e - k_j) + \varepsilon\tau_j(k_j + \rho_j h_j)$. The effect of a change in the other country's tax preference level ρ_i on this payoff can be written

$$\frac{\partial u_j}{\partial \rho_i} = \frac{\partial u_j}{\partial \tau_j}\frac{d\tau_j}{d\rho_i} + \frac{\partial u_j}{\partial \tau_i}\frac{d\tau_i}{d\rho_i} + \frac{\partial u_j}{\partial h_j}\frac{dh_j}{d\rho_i} \tag{A.65}$$

where $d\tau_j/d\rho_i$, $d\tau_i/d\rho_i$ and $dh_j/d\rho_i$ refer to the effects of first–stage changes in tax preferences on the equilibrium tax rates and number of multinational firms, as defined by equations (A.52) and (A.53), and substitution of those equations in (A.48).

Since country j chooses its own tax rate τ_i in the third stage so as to maximize its own payoff, $\partial u_j/\partial \tau_j = 0$, when the third–stage equilibrium is in Regime I, so that the first term on the right side of expression (A.65) is zero.

Since $\partial u_j/\partial h_j = \varepsilon\tau_j\rho_j$, equation (A.54) implies that the third term on the right side of expression (A.65) is also positive, if $\rho_i = 0$ initially. Lemmas A.4 and A.5 imply that $\partial u_j/\partial \tau_i > 0$ when $j \neq i$, at $\rho_i = 0$. Finally, Proposition A.2 implies that $d\tau_i/d\rho_i > 0$ at $\rho_i = 0$, completing the proof of the proposition. □

Our final result is:

Proposition A.5 *Both countries must have some tax preferences (ρ_1 and ρ_2 must be strictly positive) in any Nash equilibrium.*

Proof: The effect of a change in a country's own tax preference level ρ_i on its payoff can be written

$$\frac{du_i}{d\rho_i} = \frac{\partial u_i}{\partial \tau_i}\frac{d\tau_i}{d\rho_i} + \frac{\partial u_i}{\partial \tau_j}\frac{d\tau_j}{d\rho_i} + \frac{\partial u_i}{\partial h_i}\frac{dh_i}{d\rho_i} + \frac{\partial u_i}{\partial \rho_i} \tag{A.66}$$

Again, the first term on the right side of expression (A.66) must be zero because of the optimality of the country's own third–stage choice of τ_i ; the third term must be zero at $\rho_i = 0$ since $\partial u_i / \partial h_i = \varepsilon \tau_i \rho_i$. That means that, if $\rho_i = 0$,

$$\frac{\partial u_i}{\partial \rho_i} = (\varepsilon k_i - e) \frac{d\tau_j}{d\rho_i} + \varepsilon \tau_i h_i. \tag{A.67}$$

Lemmas A.4 and A.5 and Proposition A.2 show that the first term on the right side of (A.67) must be positive. The second term must also be positive, proving the proposition. □

An immediate corollary to these two propositions is:

Corollary 1 *Starting at $\rho_1 = \rho_2 = 0$, any coordinated small increase in tax preferences in both countries will benefit both countries.*