1. Introduction

Many explanations have been provided as to why a central government should provide (or does provide) transfers to the governments of its constituent regions. If there are hindrances to movement by people, transfers may simply be a form of redistribution from rich to poor. This rationale suggests that transfers should be unconditional, and leaves open the question of why the central government does not directly redistribute to low–income people, regardless of their region of residence.

If we believe people are mobile among regions, alternative explanations of federal transfers are needed. And many have been provided. Buchanan & Goetz (1972), and Flatters Henderson & Mieszkowski (1974) presented models in which the location decisions of migrants were not necessarily efficient, due to a mismatch of regional tax instruments with the characteristics of the public output they were required to provide. [1] Boadway & Flatters (1982) have also provided explanations of federalism along these lines.

Another justification, this time for matching grants, stems from spillovers of regional public services to residents of other regions. If regional governments ignore these benefits in their decision–making, then conditional matching grants serve as Pigouvian subsidies for the positive externality.

An alternative explanation of under–provision of regional public services, requiring compensating conditional grants, lies in competition for a mobile tax base by regional government. In the work of Wilson (1986), Zodrow & Mieszkowski (1986), and others, the necessity of a region taxing mobile capital income at source has been shown to lead to an under–provision of regional public output. One method of remedying this under–provision would be a system of matching grants from a higher level of government.

Of course there are many other normative explanations of why federal transfers should be implemented, and many positive explanations of why such transfers might persist — even if they are not justified on the grounds of efficiency or equity.

Here I present a somewhat different normative justification for transfer payments from a central government to lower levels of government. The main distinguishing features of the model are :

i : asymmetries in resource endowments among regions

ii : a distinction between "native–born" and "immigrant" residents of a region

iii : strategic behaviour on the part of regional governments

The model which I present is very similar to the one presented in Bucovetsky (1995). That model considered two regions which differed (inter alia) in per capita endowment of an immobile factor of production. [2]Strategic behaviour on the part of regional governments led to the rich region taxing labour income of all residents, and the poor region subsidizing it. In equilibrium, these taxes led to an inefficient allocation of people to regions. Too few people moved to the rich region. The appropriate form of transfers would then seem to involve the central government subsidizing the movement of people to the rich region (and from the poor region). In other words, an efficiency explanation was provided for transfers which went in exactly the opposite direction from the transfers we tend to observe.

[1] Wildasin (1986) provides an excellent analysis of the efficient matching of sub–national tax instruments with the characteristics of the public output provided.

[2] In that model, there also were differences in per capita ownership of mobile capital among residents of different regions. The strategic use of tax policy to affect the return to capital will not be considered here.

However, it turns out that central government transfers are completely neutral in the two–region model. Moving people to the resource–rich region would cause a Pareto–improvement, if side payments can also be introduced. Location–specific federal grants will move people from one region to the other, if regional policies do not adjust in response. But if regional governments alter their tax policy in response to federal grants, the effects of these grants will be completely undone in the new equilibrium.

Here I present a variant of the earlier model, and concentrate on the role of central government transfers to the regions. It is shown that the neutrality result just described is an artifact of the assumption that there are only two regions. If the number of regions is larger (and if no single region is too large), central government policy will have an effect. However, the number of regions here should not be too large. The case for federal transfers still rests on the strategic behaviour of regional governments. If no region is large enough to have any market power, then no region's government will create any impediments to mobility.

The main concern of this paper is how people should be re–allocated among regions. The point mentioned above — that strategic behaviour by regional governments induces too little migration — continues to hold when there are more than two regions. Therefore, the model presented here provides a justification for location–specific subsidies, but subsidies which should be higher for resource–rich regions.

Of course, the idea that transfers to poor regions impede efficient migration patterns is not a new one. But in this model the strategic behaviour of regional governments means migration will not be efficient without equalization. Moreover, this inefficiency cannot be cured by providing regional governments with a better mix of tax instruments. Locational subsidies seem the most plausible corrective device for the inefficiency of equilibrium.

This leaves open the question of why actual transfers go in the wrong direction. The only contribution this paper makes to this positive debate is a brief analysis of who gains, and who loses, from federal government transfers.

2. The Basic Model

I assume that a country is divided into I regions. This governmental structure is taken as given. In this section, the strategic behaviour of each region's government will be analyzed. But the policies of the central government will be taken as given. The central government's decision making will be discussed in the subsequent sections.

Thus the players in the model are assumed to move in the following sequence : First the central government commits to a set of regional grants, and commits to finance the grants using some tax instruments. Next, the region's governments choose their fiscal instruments, having observed the central government's policy choice. The regions' governments move simultaneously. Finally, the individual residents of the country choose a region in which to reside (having observed the central and regional governments' policies, and all moving simultaneously). The solution concept is sub–game perfect Nash equilibrium, although no game–theoretic issues of any sophistication will be explored here.

There is a single homogeneous output, produced in all the regions. The two inputs to production are labour, and resources. [3]Resources are perfectly immobile among regions. The people who supply the labour

[3] Other inputs to production could be allowed, without changing any of the results of this section, if the inputs were of either of the following types. First, they could be perfectly immobile among regions. Second, they could be perfectly mobile, not only among regions, but internationally, with the country in question being a small enough part of the world market that

services can move costlessly among the regions. For the moment, the production function for region i will be written $F_i(L_i)$, a function of the aggregate labour supply in the region, with differences in resource endowment subsumed in the form of the production function. A more specific assumption about how productivity varies among regions will be made subsequently. Output is assumed to be an increasing concave function of the aggregate labour input.

$$F_i'(L_i) > 0 \quad ; \quad F_i''(L_i) < 0 \quad i = 1, 2, \ldots, I$$

Each region i has N_i "native–born citizens". Here place of birth is not the same as place of residence. People are citizens of the region in which they were born, and cannot alter their citizenship. The immobile factor in a region is owned (equally) by all the citizens of the region. Despite the costless mobility, citizens of a region do not have the same interests as immigrants. The citizens own assets in the region itself ; immigrants own assets elsewhere. The return to a citizen's resource ownership is her share in the aggregate value of output produced in the region, net of labour costs and any applicable taxes. Each of the N_i citizens of region i gets a share $1/N_i$ of the return to this immobile input.

A crucial assumption is that citizens choose a region's policies. Perhaps this is a constitutional provision ; only citizens vote. [4] Or the assumption may be justified by assuming that migration flows are small enough that immigrants are a minority in each region. Finally it may be motivated by recognizing that the local resource owners have a greater vested interest in influencing government policy. The return to resources in the region is influenced more by the regional government's policy than is the return to resources in other jurisdictions. People should specialize in lobbying and other political activity in the region which can most influence the value of their assets.

The government of each region provides a public output, which is produced using a "pure private good" technnology. It uses the output of the private sector as an input, and units are chosen so that each unit of the public sector output uses one unit of the private good as input.

The two regional tax rates are a unit tax at the rate t_i per worker on labour income, and a unit tax at the rate τ_i per citizen. This latter tax is exactly equivalent to a tax on the earnings of the immobile factor, given that citizens own the immobile factor. Therefore the government budget constraint for region i is

$$t_i L_i + \tau_i N_i - g_i L_i \geq 0 \tag{1}$$

where L_i is the number of workers resident in region i, and g_i is the quantity provided of the public sector output.

Workers are the same as residents, and each worker–resident provides her labour inelastically. [5] The utility function of all residents of all regions has the same form :

$$u(x, g) = x + \gamma(g)$$

the world return to the input could be taken as fixed.

[4] And new immigrants do not obtain citizenship.

[5] The assumption of inelastic labour supply means that the regional government can discriminate perfectly against immigrants. Allowing them to charge an admission fee to immigrants would not change their policy, since the same effect can be produced by rasing the labour tax and lowering the citizenship tax. Thus the inefficiency of the equilibrium would not disappear if governments could discriminate explicitly against immigrants. It would disappear if governments could discriminate *among* immigrants.

where x is the person's consumption of private goods, and g is the level of local public sector output provided in the region in which she resides. The results do depend on preferences being representable in this quasi–linear fashion. [6]

In addition, the federal government may provide a regional locational subsidy s_i to all residents of region i. This subsidy is financed by a national tax, which may vary in its rate for *citizens* of different regions. In particular, ζ_i denotes the share of the federal taxes paid by citizens of region i. All citizens of a region are assumed to pay the same share of federal government taxes. [7]

Perfect competition in the private sector ensures that workers are paid the value of their marginal product. This means a worker resident in region j receives $F_j'(L_j) - t_j + s_j$ in net labour income, in addition to her earnings based on her citizenship. Her overall net–of–tax income, if she is a citizen of region i, is

$$x_{ij} \equiv F_j'(L_j) - t_j + s_j + \frac{1}{N_i}\big[F_i(L_i) - F_i'(L_i)L_i - \zeta_i \sum_k s_k L_k\big] - \tau_i$$

Since people are perfectly mobile among regions, they locate in the region j which yields the highest value of $u(x_{ij}, g_j) = x_{ij} + \gamma(g_j)$. That means citizens of region i choose to reside in the region j yielding the highest value of

$$F_j'(L_j) - t_j + s_j + \gamma(g_j)$$

which is independent of their region of birth i, thanks to the quasi–linear utility function. If all regions are populated, then, a consequence of the mobility is

$$F_i'(L_i) - t_i + s_i + \gamma(g_i) - F_j'(L_j) + t_j - s_j - \gamma(g_j) = 0 \qquad \text{all} \quad i, j \tag{2}$$

and the expression for the private consumption of a citizen of region i can be written

$$x_i = F_i'(L_i) - t_i + s_i + \frac{1}{N_i}\big[F_i(L_i) - F_i'(L_i)L_i - \zeta_i \sum_k s_k L_k\big] - \tau_i$$

Citizens of region i wish to maximize their utility, and so can be regarded as maximizing $x_i + \gamma(g_i)$ subject to equations (1) and (2) and the adding–up requirement

$$\sum_k L_k = \sum_k N_k \tag{3}$$

They can be regarded as choosing t_i, τ_i, g_i and each of the L_k's as instruments for this constrained maximization. In doing this maximization, they treat each of the other regions' wage tax rates t_j and per capita public expenditure levels g_j as given. Notice that they cannot regard all three of any other region's fiscal instruments t_j, τ_j and g_j as fixed ; the budget constraint (1) for region j shows that at least one of these instruments must change if people move in response to some policy change in region i. I assume that regions regard t_j and g_j as strategic variables and the source–based tax τ_j as a residual which adjusts so as to satisfy the government budget constraint.

This choice of which strategic variables to regard as given does affect the Nash equilibrium, as emphasized in Wildasin (1988). The choice made here was motivated by the fact that other regions' source–based tax

[6] Quasi–linearity implies that in equilibrium all citizens of all regions will be indifferent among all regions as places of residence.

[7] If some federal taxes depended on the region of residency, rather than the region of citizenship, these taxes could simply be subtracted from the location subsidies.

rates τ_j do not appear directly in the constraints faced by region i, while the other two fiscal variables do, since they affect people's migration decisions.

The Lagrangean for region i's citizens is

$$F_i(L_i) + F_i'(L_i)(N_i - L_i) - \tau_i N_i - t_i N_i + \gamma(g_i)N_i + s_i N_i - \zeta_i[\sum_k s_k L_k]$$
$$+\lambda[t_i L_i + \tau_i N_i - g_i L_i]$$
$$+\sum_{j\neq i} \mu_j [F_i'(L_i) - t_i + s_i + \gamma(g_i) - F_j'(L_j) + t_j - s_j - \gamma(g_j)]$$
$$+\nu[\sum_{k=1}^{I} L_k - \sum_{k=1}^{I} N_k]$$

This Lagrangean is maximized with respect to t_i, τ_i, g_i and each of the L_j's, treating the other regions' t_j's and g_j's as given. The first–order conditions for maximization of the above Lagrangean are

$$F_i''(L_i)[N_i - L_i] + \lambda(t_i - g_i) + (\sum_j \mu_j)F_i''(L_i) + \nu - \zeta_i s_i = 0 \tag{4.i}$$

$$-\mu_j F_j''(L_j) + \nu - \zeta_i s_j = 0 \qquad j \neq i \tag{4.j}$$

$$-N_i + \lambda L_i - \sum_j \mu_j = 0 \tag{4.t}$$

$$-N_i + \lambda N_i = 0 \tag{4.τ}$$

$$\gamma'(g_i) - \lambda L_i + (\sum_j \mu_j)\gamma'(g_i) = 0 \tag{4.g}$$

The first–order conditions on the tax rates imply $\lambda = 1$ and $\sum \mu_j = L_i - N_i$, which then imply, from the first–order condition on the public expenditure g_i per capita, that

$$\gamma'(g_i) = 1 \tag{5}$$

which is just the Samuelson condition for the first–best level of public output provision, that the sum of people's marginal rates of substitution equal the the aggregate marginal rate of transformation. This equation means that all regions provide the same level of public expenditure per capita, which will be denoted g^*. Only differences in the net wage $F_j'(L_j) - t_j + s_j$ will matter for location decisions.

Equation (5) means that there is no problem with the public sector here. The availability of both a tax on mobile labour (t_i) and a head tax on citizens (τ_i) gets rid of the under–provision problem, which arises in models of tax competition because regions do not have a non–distortionary residence–based tax instrument, as they do here.

Another way to see the efficiency–of–public–good–supply result is to suppose that $g_i < g^*$, so that $\gamma'(g_i) > 1$. Then suppose that region i increased g_i slightly. The assumption that $\gamma'(g_i) > 1$ means that the region's government could raise t_i by the change in $\gamma(g_i)$, so that the same population L_i as before satisfied the mobility constraint (5), and raise more income from the change in the labour tax than the change in

g_i cost. That is, mobile immigrants are willing to pay more than the cost of the increment in public sector output. That would enable the government to lower the head tax τ_i, raising its citizens' incomes.

Since the choice of public expenditure by the region can be ignored henceforth, the maximization problem can be presented in a less daunting form than the above Lagrangean. Since

$$\tau_i N_i = g^* L_i - t_i L_i$$

when the regional government budget constraint is satisfied with equality, the government can be viewed as choosing its labour income tax rate t_i to maximize

$$W_i \equiv F_i(L_i) + F_i'(L_i)(L_i - N_i) - (g^* - t_i)L_i - tN_i + \gamma(g^*)N_i - \zeta_i \sum_{k=1}^{I} s_k L_k \tag{6}$$

where the population L_i of each region is determined by the vector of the tax rates $(t_1, t_2, \ldots, t_I)$. Equation (6) makes the total income W_i of a region's citizens depend on only a single choice variable (as well as the other regions' tax rates t_j), when this dependency of L_i on the vector of tax rates is taken into account. The first–order condition for the citizens of region i now can be written

$$\frac{\partial W_i}{\partial t_i} = [F_i''(N_i - L_i) - (g^* - t_i)]\frac{\partial L_i}{\partial t_i} + (L_i - N_i) - \zeta_i[\sum_{k=1}^{I} s_k \frac{\partial L_k}{\partial t_i}] \tag{7}$$

To get the effect of tax rates on regional population, write the system of mobility conditions (2), along with the population adding–up constraint, as the following system

$$F_i'(L_i) - t_i + s_i = y^m \qquad i = 1, 2, \cdots, I \tag{8}$$

$$\sum_{i=1}^{I}(L_i - N_i) = 0 \tag{3}$$

Differentiation of this system with respect to the population levels, the "mobile income" y^m, and any one of the tax rates (say t_1) yields

$$\begin{pmatrix} -F_1'' & 0 & 0 & \cdot & \cdot & 0 & 1 \\ 0 & -F_2'' & 0 & \cdot & \cdot & 0 & 1 \\ 0 & 0 & -F_3'' & \cdot & \cdot & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & -F_I'' & 1 \\ 1 & 1 & 1 & \cdot & \cdot & 1 & 0 \end{pmatrix} \begin{pmatrix} dL_1 \\ dL_2 \\ dL_3 \\ \cdot \\ \cdot \\ dL_I \\ dy^m \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \end{pmatrix} dt_1 \tag{9}$$

By adding $1/F_i''$ times each of the first I rows of the above matrix to the final row, the determinant can be computed as

$$\Delta = (\prod_{i=1}^{I}[-f_i''])(\sum_{i=1}^{I}[1/F_i''])$$

For any allocation of people denote

$$A_i \equiv -1/F_i''(L_i) > 0$$

which is the derivative of a region's wage rate with respect to its population. Let

$$A \equiv \sum_{j=1}^{I} A_j$$

$$a_i \equiv A_i/A$$

so that each of the a_i's is positive, and the a_i's sum to 1.

The variable a_i will be referred to as the "relative wage responsiveness", since the partial derivative of the wage with respect to the labour force in a region is $-1/F_i''$.

Since $\partial L_1/\partial t_1$ is the determinant of the matrix obtained by crossing out the first row and column of the above matrix, times -1, divided by Δ, therefore

$$\frac{\partial L_1}{\partial t_1} = -A_1(1-a_1) < 0$$

By analogy, then

$$\frac{\partial L_i}{\partial t_i} = -a_i(1-a_i)A \tag{10}$$

for any region i. The equation $F_i'(t_i) - t_i + s_i = F_j'(t_j) - t_j + s_j$ for any pair of regions i and j provides the cross–regional effects of tax changes,

$$\frac{\partial L_j}{\partial t_i} = a_i a_j A > 0 \quad i \neq j \tag{11}$$

Substituting equations (10) and (11) into the first–order condition (7),

$$\frac{\partial W_i}{\partial t_i}\frac{1}{a_i} = (L_i - N_i) + A(1-a_i)(g^* - t_i) + \zeta_i s_i A - \zeta_i A \sum_{j=1}^{I} a_j s_j$$

If the tax rate t_i on mobile labour is set optimally, then

$$(L_i - N_i) + A(1-a_i)(g^* - t_i) + \zeta_i s_i A - \zeta_i A \sum_{j=1}^{I} a_j s_j = 0 \tag{12}$$

Equation (12) is the main characterization of the Nash equilibrium (in the sub–game played by regional governments). The remainder of the paper uses this characterization to examine how differences in regions' population and resource endowments affect their policies, and how the central government can alter those policies.

3. A Role for the Central Government

Suppose first that there were no federal government, so that each of the location–based grant rates s_k were zero. If all regions were identical, and the Nash equilibrium were symmetric, then L_i would equal N_i since there would be no net migration. Then equation (12) would imply $t_i = g_i$. That means that each region would levy the same tax on workers, and no source–based taxes. The resulting allocation of people would be efficient, since every person's marginal product would be the same in each region. In the absence of a federal government, differences between regions are necessary in order for the Nash equilibrium not to be efficient.

On the other hand, if there is any migration at all in equilibrium, then equation (12) says that, in the absence of federal transfers, at least one region must have $t_i > g^*$ and at least one region must have $t_i < g^*$. Since perfect mobility implies $F_i'(L_i) - t_i = F_j'(L_j) - t_j$, and efficiency requires $F_i'(L_i) = F_j'(L_j)$, then the Nash equilibrium cannot be efficient.

My earlier paper provides a more detailed proof (for the case $I = 2$) that Nash equilibrium is efficient if and only if regions are identical if and only if there is no migration in equilibrium. That result is repeated in appendix 1 of this paper. Identical productivity among all regions — in the sense that $F_i'(N_i) = F_j'(N_j)$ — is necessary and sufficient for the Nash equilibrium to be efficient in the absence of federal transfers. And the Nash equilibrium in the absence of those transfers is efficient if and only if net migration to any region is 0.

If regions are not identical (and there are no federal grants), then the "rich" regions, whose high endowment per capita of immobile factors attract immigration, will tend to exploit that immigration by charging residence–based taxes which exceed the cost per capita of the public sector. Analogously, regions with net emigration will tend to levy source–based taxes, and "penalize" emigrants. Strategic tax–setting impedes migration, in that the marginal product of workers is too high in regions receiving net immigration and too low in regions with net emigration. [8]

If there are only two regions, then equation (12) implies that at any equilibrium

$$(L_1 - N_1) + A(1 - a_1)(g^* - t_1 + \zeta_1[s_1 - s_2]) = 0 \tag{13.1}$$

and

$$-(L_1 - N_1) + Aa_1(g^* - t_2 - (1 - \zeta_1)[s_1 - s_2]) = 0 \tag{13.2}$$

where I have used the facts that $L_2 - N_2 = -(L_1 - N_1)$, $1 - a_2 = a_1$, and $\zeta_2 = (1 - \zeta_1)$ when there are only two regions.

Suppose that $(t_1, \tau_1, g_1, L_1, t_2, \tau_2, g_2, L_2)$ is a Nash equilibrium when there is no federal government. Then equations (13.1) and (13.2) will also be satisfied by $(t_1', \tau_1', g_1, L_1, t_2', \tau_2', g_2, L_2)$ with

$$t_1' = t_1 + \zeta_1(s_1 - s_2)$$

$$t_2' = t_2 - \zeta_2(s_1 - s_2)$$

$$\tau_i' N_i = g_i L_i - t_i' L_i \qquad i = 1, 2$$

when the federal government introduces location–based subsidies (s_1, s_2). The construction of the new fiscal variables ensures that each region's government's budget constraint (1) will be satisified. Thus if the new variables satisfy the mobility constraint (2), then they constitute a Nash equilibrium when the federal government introduces location–based subsidies.

Since $t_1' - t_2' = t_1 - t_2 + (\zeta_1 + \zeta_2)(s_1 - s_2)$, the fact that the shares of the federal tax must add up to 1 implies that the change in $t_1 - t_2$ exactly equals the change in $s_1 - s_2$. Any increased attraction in region 2 as a residence due to higher federal grants is exactly offset by an increase in the regional wage tax difference. If the original vector of fiscal variables constituted a Nash equilibrium in the absence of federal grants, then the new vector will be a Nash equilibrium — with the same allocation of residents — when the federal government introduces the location–based subsidies.

[8] This characterization is discussed in the next section of the paper.

But this result implies federal grants are neutral when there are only two jurisdictions. The grants have not affected the misallocation of residents between regions, although they may have altered the distribution of income between citizens of the two regions.

The neutrality of federal government grants holds only when there are two regions in the country. When there are three or more regions, the polar opposite case emerges.

THEOREM 1 : If no region contains more than half the national tax base, then any feasible allocation of people to regions can be sustained as a Nash equilibrium, if the federal government chooses its location–specific grants $(s_1, s_2, \ldots, s_I)$ suitably.

PROOF : Let $(L_1, L_2, \ldots, L_I)$ be any feasible allocation of people to regions. Equation (5) shows that in any Nash equilibrium all regions will provide the same level g^* of public expenditure per capita. Then the tax rates $(t_1, t_2, \ldots, t_I)$ and the given allocation $(L_1, L_2, \cdots, L_I)$ of people will form a Nash equilibrium for the subsidy vector $(s_1, s_2, \ldots, s_I)$ if equation (12) holds for every region i, and the mobility condition

$$F_i'(L_i) - t_i + s_i + g^* - F_1'(L_1) + t_1 - s_1 - g^* = 0 \tag{14}$$

holds for every region $i > 1$.

Suppose that the subsidies to each region were increased by the same constant ϵ. If none of the t_i's were changed, then the value of the left–hand side of equation (12) would not change, since the a_j's there sum to 1. Moreover, the mobility condition (2) would still be satisfied. Therefore, increasing (or decreasing) all the s_i's by the same constant will not affect the equilibrium tax rates, or the equilibrium allocation of people to regions. Without loss of generality, one of the subsidy rates, s_1, may be set equal to 0. [9]

The government can achieve the given allocation of people $(L_1, L_2, \ldots, L_I)$ as a Nash equilibrium if there exists a vector $(s_2, s_3, \ldots, s_I, t_1, t_2, \ldots, t_I)$ which satisfies equations (12) for all i, and equations (14) for all $i > 1$.

Here the arbitrary allocation of people to regions is being taken as given, and the central government's problem is to find a vector $(s_2, s_3, \ldots, s_I, t_1, t_2, \ldots, t_I)$ which satisfies these equations. These equations are linear in the tax and subsidy rates, and there are $2I - 1$ equations, corresponding to the $2I - 1$ unknowns. A sufficient condition for the central government to be able to achieve the allocation is for the system of equations to be linearly independent.

If the new variable δ_i is introduced,

$$\delta_i \equiv t_i - s_i \qquad i > 1$$

then equations (12) and (14) become

$$\delta_i - t_1 = F_i'(L_i) - F_1'(L_1) \qquad i = 2, 3, \cdots I \tag{15}$$

$$\sum_{j=2}^{I} a_j[\zeta_1(\delta_j - t_j) - t_1] = \frac{1}{A}(N_1 - L_1) - (1 - a_1)g^* \tag{16.1}$$

$$\sum_{j \neq i, j \neq 1} a_j[\zeta_i(t_i - \delta_i - t_j + \delta_j) - t_i] + a_1[\zeta_i(t_i - \delta_i) - t_i] = \frac{1}{A}(N_i - L_i) - (1 - a_i)g^* \qquad i > 1 \tag{16.i}$$

[9] Increasing all the locational subsidies by a constant will affect the incomes of citizens of different regions, in general. But here I am concerned only with the sustainability of an allocation of people to regions.

These equations are now linear in the δ_i's and t_i's. Therefore, a solution will exist, for the given right–hand side of the equations, if the matrix of the coefficients of the δ_i's and t_i's is non–singular. This matrix is

$$\begin{pmatrix} 1 & 0 & \cdot & \cdot & 0 & -1 & 0 & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & 0 & -1 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 1 & -1 & 0 & \cdot & \cdot & 0 \\ \zeta_1 a_2 & \zeta_1 a_3 & \cdot & \cdot & \zeta_1 a_I & -(1-a_1) & -\zeta_1 a_2 & \cdot & \cdot & -\zeta_1 a_I \\ -\zeta_2(1-a_2) & \zeta_2 a_3 & \cdot & \cdot & \zeta_2 a_I & 0 & -(1-\zeta_2)(1-a_2) & \cdot & \cdot & -\zeta_2 a_I \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \zeta_I a_2 & \zeta_I a_3 & \cdot & \cdot & -\zeta_I(1-a_I) & 0 & -\zeta_I a_2 & \cdot & \cdot & -(1-\zeta_I)(1-a_I) \end{pmatrix}$$

If the sum of the first $(I-1)$ columns is added to the I–th column, then the determinant of this matrix is unchanged, and (since the upper–right I–by–I sub–matrix now contains nothing but zeroes), equals the determinant of

$$\begin{pmatrix} -(1-\zeta_1)(1-a_1) & -\zeta_1 a_2 & \cdot & \cdot & -\zeta_1 a_I \\ -\zeta_2 a_1 & -(1-\zeta_2)(1-a_2) & \cdot & \cdot & -\zeta_2 a_I \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -\zeta_I a_1 & -\zeta_I a_2 & \cdot & \cdot & -(1-\zeta_I)(1-a_2) \end{pmatrix}$$

Since the ζ_i's and the a_i's are all between 0 and 1, and all sum to 1, all the entries in the matrix are non–positive. The sum of the off–diagonal elements in the i–th row of the matrix is $-\zeta_i(1-a_i)$, which means that the matrix has a dominant diagonal if $(1-\zeta_i) > \zeta_i$, or $\zeta_i < 0.5$. And matrices with strictly dominant diagonals are non–singular. [10]

[Note that when $I = 2$, it cannot be the case that both regional tax shares ζ_i are less than one–half. The fact that $1 - a_1 = a_2$ and $1 - \zeta_1 = \zeta_2$ then show Z' must be singular, as the previous neutrality result required.]

4. Haves and Have–Nots

In most of what follows, all differences between regions' production functions will be ascribed to differences in per capita endowments of an immobile factor, "resources". Thus it will be assumed that

$$F_i(L_i) \equiv F(L_i, R_i)$$

where the function $F(\cdot,\cdot)$, which does not differ among regions, exhibits constant returns to scale, and diminishing marginal product for each input. Because of the constant returns to scale,

$$F(L_i, R_i) = F(\frac{L_i}{R_i})R_i \equiv f(\lambda_i)R_i$$

so that

$$F_i'(L_i) = f'(\lambda_i)$$

and

$$F_i''(L_i) = \frac{f''(\lambda_i)}{R_i}$$

[10] See Gale and Nikaido (1965), for example.

Under this assumption, a "have" region is one which has a high endowment of resources per citizen. A fairly straightforward implication of equation (12) is that "have" regions levy high labour income taxes, but get too few residents, in the absence of corrective central government transfers.

THEOREM 2 : Suppose there are no central government transfers. Then there is a "cut–off" level of the resource endowment R_i/N_i such that, in equilibrium, all regions whose endowment per citizen is above the cutoff levels receive net immigration ($L_i > N_i$) and levy a labour income tax greater than g^*. All regions whose resource endowment per citizen is below the cut–off provide net emigration, and levy a labour income tax less than g^*.

PROOF : If equation (12) is added up over all regions i, when there are no transfers, then the fact that net immigration must sum to zero implies that a weighted average of the regions' tax rates must equal g^*. (Here the weights are proportional to $(1 - a_i)$.)

Equation (12) also indicates that net immigration is positive if and only if $t_i > g^*$. So the relationship between high taxes and net immigration is immediate. What must be proved is that high–tax regions are those with high resource endowment per citizen.

Suppose that $t_i > g^* > t_j$. Then the mobility condition (2) implies $F_i'(L_i) > F_j'(L_j)$. Under the assumption made about technology, this inequality implies

$$\frac{L_i}{R_i} < \frac{L_j}{R_j}$$

Since $t_i > g^* > t_j$, it is also the case that

$$L_i - N_i > 0 > L_j - N_j$$

so that

$$\frac{N_i}{R_i} < \frac{L_i}{R_i} < \frac{L_j}{R_j} < \frac{N_j}{R_j}$$

which implies any region with $t_i > g^*$ must have a higher resource endowment per capita than any region with $t_j < g^*$, completing the proof of the theorem.

The implication of the theorem is that "have" regions "exploit" immigrants by taxing their incomes (and use the proceeds to pay a subsidy to citizens). These taxes reduce immigration from its efficient level — that which equalizes the gross wage among regions. But they do not reverse the direction of immigration — from resource–poor regions to resource–rich regions.

The above theorem only splits regions into two groups. It does not provide a complete ordering of regions by tax rates, or by levels of immigration. If regions all have the same aggregate resource endowment, but differ in the number of citizens who own the resources, then such an ordering can be provided, if the sign of $\partial W_i/\partial t_i$ provides an indication of how the region's tax rate should be set.

ASSUMPTION *A* : If $\partial W_i/\partial t_i > \partial W_j/\partial t_j$ whenever $t_i = t_j$, then region i sets a higher equilibrium tax rate than region j.

This condition holds if a region's reaction function — expressing its optimal tax rate as a function of another region's tax rate, holding constant all other region's tax rates — has a slope less than 1 in absolute value, and if each region's citizens' income is a quasi–concave function of its own tax rate. If the endogeneity of a_i is ignored, then

$$\frac{\partial^2 W_i}{\partial t_i^2} = -a_i A(1-a_i)(1+a_i) < 0$$

and

$$\frac{\partial^2 W_i}{\partial t_i \partial t_j} = a_i^2 a_j A$$

so that the assumption must hold. It would only be violated if the effects of tax rates on a_i (working through changes in the second derivative of F_i) were of the "wrong" sign, and of large magnitude.

The assumption will be used in several results in the paper. It means that whether one region levies a higher tax than the other can be checked by looking at equation (12) alone.

THEOREM 3 : If two regions have the same resource endowment R_i, and if assumption A holds, then the region with fewer citizens levies a higher labour tax rate, and has a smaller population in equilibrium, when there are no central government transfers.

PROOF : Given the assumption that R_i is the same for all regions, if $t_i = t_j$, then $L_i = L_j$, and $a_i = a_j$. Then

$$\frac{\partial W_i}{\partial t_i} - \frac{\partial W_j}{\partial t_j} = N_j - N_i$$

if $t_i = t_j$, proving $t_i > t_j$ if and only if $N_i < N_j$. The mobility condition (2) then implies $F_i'(L_i) > F_j'(L_j)$, implying $L_i < L_j$.

The above theorem gives some indication that higher resource endowment per capita leads to higher taxes. The next result looks at the effect of regional size alone, holding constant the resource endowment per citizen. Here there is a "magnification effect". Larger "have" regions levy higher taxes, while larger "have–not" regions levy lower taxes.

THEOREM 4 : If $R_i/N_i = R_j/N_j$ and $R_i > R_j$ then $t_i > t_j$ if both regions are "have" regions, and $t_i < t_j$ if they are both "have–not" regions (when there are no central government transfers).

PROOF : From equation (12), $\partial W_i/\partial t_i - \partial W_j/\partial t_j$ is proportional to

$$(R_i - R_j)[(\frac{L_i}{R_i} - \frac{N_i}{R_i}) - \frac{1}{(-f'')}(g^* - t_i)]$$

when $t_i = t_j$.

If $t_i = t_j = t$, then $L_i/R_i = L_j/R_j$, so that $\partial W_i/\partial t_i > \partial W_j/\partial t_j$ if and only if

$$[\frac{-1}{(-f'')}](R_i - R_j)(g^* - t)$$

5. The Effects of Federal Grants

In this model, the job of federal transfers is to undo the inefficiency caused by strategic behaviour on the part of regional governments. That behaviour involves resource–rich regions reducing immigration through high residence–based taxes, and resource–poor regions reducing emigration through low residence–based taxes.

Since each region provides its public output efficiently (thanks to the availability of the source–based resource income tax), the only efficiency issue is the allocation of workers across regions. National income is maximized if gross wages are the same in every region. If the federal government is free to vary the (lump–sum) taxes which finance its transfers, then any system of transfers which increases national income can induce a Pareto–improvement. Section 3 showed that (when no single region was too large) for **given** regional shares ζ_i in the federal tax burden, any feasible allocation of people to regions could be achieved through some set of federal transfers s_i. If the distributional implications of the re–allocation are unappealing (for example, if the grants do not induce a Pareto–improvement), then the tax shares ζ_i could be altered.

Any transfer system which induces movement to the region with the highest wage, and movement from all other regions, must raise national income. The change in national income is

$$\sum_{i=1}^{I} F_i'(L_i) dL_i$$

which equals

$$\sum_{i=2}^{I} [F_i'(L_i) - F_1'(L_1)] dL_i$$

if the total number of citizens in the nation does not change. This expression will be positive if region 1 has the highest gross wage, and if each other region experiences a net outflow of workers in response to the policy change.

Naturally, it might be expected that increasing s_i, the lump–sum federal payment to each worker resident in region i, should induce immigration to that region. And that certainly would be the case if the taxes levied by each region's government did not change. However, here the tax rates are endogenous. Changes in the pattern of grants, and in the federal taxes which pay for them, induce changes in regional tax rates. The last term (involving the sum of the $a_j s_j$'s) in equation (12) represents the effects of the federal taxes. Regions' incentives, other things equal, are to move workers to regions with low federal transfers, thus reducing their own share of federal tax burden. Notice that this effect is proportional to ζ_i, the share of the region in the federal tax burden.

Suppose for example that the Canadian federal government payments to the Atlantic provinces are made proportional to the provinces' populations (and that these grants per capita are higher than in other provinces). If the grants are paid for out of general federal tax revenues, then any immigration to the Atlantic provinces from provinces with lower federal per capita payments will raise federal taxes. Since Ontario residents pay some share of those taxes, the Ontario government has some incentive to lower taxes, to attract immigration and reduce these federal taxes.

The question then, is whether, when these strategic considerations are taken into account, an increase in s_i still will induce immigration to region i. Here the answer is yes — at least when third and higher derivatives of the production function can be ignored. That is, the relative wage responsiveness variables a_i will be treated here as constants. This treatment is strictly correct only if the production function is quadratic — or if all the coefficients on any a_i terms in equation (12) are zero. [11]

ASSUMPTION B : All the relative wage responsiveness variables a_i are constant.

[11] This latter possibility emerges if there are no grants initially, and if the initial equilibrium is efficient.

Assumption B implies assumption A. Under the assumption of identical technologies, it also implies that the a_i's equal the regions' share of the immobile resources.

Note first from equation (8) that the effect of a grant on population distribution, holding regional fiscal policy constant, is exactly the opposite of the effect of the wage tax, so that

$$\frac{\partial L_i}{\partial s_i} = (1 - a_i)A \tag{17}$$

$$\frac{\partial L_j}{\partial s_i} = -a_i a_j A \quad j \neq i \tag{18}$$

where the derivatives are understood not to incorporate the strategic reactions of regions governments.

To evaluate these reactions, the first–order conditions (12) of each region can be differentiated with respect to the tax rates t_j of all regions, and the locational grants, to get

$$(1 - a_i)(1 + a_i)dt_i - \sum_{j \neq i} a_i a_j dt_j = (1 - a_i)(a_i + \zeta_i)ds_i - \sum_{j \neq i} a_j(a_i + \zeta_i)ds_j \tag{19}$$

when the a_j's are treated as constants.

Equation (19) provides a fairly straighforward system of I linear equations in the I endogenous tax rates. Some tedious matrix algebra, deferred to appendix 2, proves

THEOREM 5 : If the share ζ_i of region i's citizens in federal taxes is less than 1, then

$$1 > \frac{\partial t_i}{\partial s_i} > 0$$

$$\frac{\partial t_j}{\partial s_i} < 0 \quad j \neq i$$

If no region's share of resources is more than one half, then an increase in s_i must induce increased immigration to region i in the new equilibrium.

PROOF : see the appendix

When the share ζ_i of the federal tax paid by citizens of region i is 1 (and all other regions' citizens' shares are 0), then an increase in s_1 is exactly the same as a decrease in t_i. Any attempt to alter the distribution of residents in this way will be completely offset by an equal increase in t_i by region i's citizens (with no changes in any other regions' tax rates).

But as long as there is some sharing of the cost of the location subsidies among regions, then — if no single region is too large — the strategic behaviour of the regional governments will not thwart the basic effect of the subsidies. Increasing the subsidy for location in region i will move people to the region.

The theorem (which has been proved only under the rather strong assumption B) does not guarantee that the subsidy will decrease the population of every other region, nor does it even guarantee that the region to which the subsidy has been increased will receive the largest net immigration. But it does imply that the gross wage will fall more in that region than in any other.

Throughout the paper, it has been assumed that all citizens of a region are identical. I cannot talk of "workers", or "resource owners" separately here. But I can divide each person's income into her "immobile income" from factor ownership,

$$N_i y_i^0 \equiv F_i(L_i) - F_i'(L_i)L_i - \tau_i N_i - \zeta_i \sum_{k=1}^{I} s_k L_k$$

and her "mobile income"

$$y^m = F_i'(L_i) - t_i + s_i$$

which is the same in all regions in equilibrium.

Theorem 5 says that any increase in s_i must lower the wage tax rates in all other regions, and it must increase the gross wage in at least one region j (since the population of region i must increase). That means that

COROLLARY : Under the assumptions of theorem 5, an increase in any region's location subsidy must raise mobile income y^m.

If an increase in a subsidy moves workers the "wrong" way, then it must lower national income. Therefore, an increase in a location subsidy to a region with a low gross wage will still increase everyone's mobile income, but will also lower the immobile income of citizens of some region.

Part of this result stems from the assumption that the federal transfer programme is funded by a tax on the immobile income in various regions. But if the cost of the programme is ignored, then

$$dy_i^0 = (t_i - g^*)dL_i + L_i dt_i - F_i''(L_i)dL_i$$

If region i did not get the subsidy increase, then theorem 5 says that $dt_i < 0$. If the region also were a "have" region which lost population, then all 3 terms in the above expression would agree. The immobile income of citizens of the region would fall, even if they did not have to pay for any of the federal subsidy.

The proof of theorem 5 shows that the equilibrium tax rates are linear in the subsidy rates s_i. Specifically,

$$t_i = t_i^0 + (a_i + \zeta_i)(s_i - a \cdot s) - \frac{(a + \zeta) \cdot a}{m} a_i(a \cdot s) + \frac{a_i}{m} \sum_{k=1}^{I} a_k(a_k + \zeta_k)s_k \tag{20}$$

where

$$m \equiv 1 - (a \cdot a)$$

and s, ζ and a are the vectors of subsidy rates, federal tax burden shares and resource shares.

Equation (20) shows the effect of a change in a region's share of the federal tax burden on the equilibrium tax rate :

$$\frac{\partial t_i}{\partial \zeta_i} = (s_i - a \cdot s)(1 + \frac{a_i^2}{m}) \tag{21}$$

$$\frac{\partial t_i}{\partial \zeta_j} = (s_j - a \cdot s)\frac{a_i a_j}{m} \quad j \neq i \tag{22}$$

Of course a single tax burden share ζ_i cannot be changed in isolation. But these results show increasing the share of the tax burden born by regions with high subsidy rates will increase wage tax rates in all regions. Also, not surprisingly, the "direct" effect of the tax change induced by an increase in ζ_i will be much larger in magnitude than the indirect effect on other regions' wage tax rates.

These results suggest that making the "have" regions' citizens pay for a larger share of subsidies will to some degree attenuate the effects of those subsidies, if they are directed at moving residents to the "have" regions. Of course theorem 1 shows that any allocation can be achieved with any distribution of the share of the federal tax burden, so that this attenuation is not fatal.

The results also suggest that making rich regions pay a large share of transfers in the "wrong" direction may reduce the adverse efficiency consequences of those grants.

6. Lacunae, Conclusions, Extensions

The model presented here has several positive implications for how regions differ, in the absence of grants from a central government. It implies that gross wages will not be equalized by migration. Rich regions which attract migration will still have higher wages. These higher wages will be offset by higher taxes.

The wage tax is not the only tax is the model presented here. There are also the taxes on "citizens". Regions with high wage taxes levy low taxes on citizens. (In fact $\tau < 0$ for all regions receiving net immigration in equilibrium.) Thus a somewhat more accurate implication of the model than "rich regions have high tax rates" is that "rich regions have higher tax rates on mobile factors, and lower tax rates on immobile factors" than do poorer regions.

The model suggests that most countries' regional policies are wrong, given that they tend to subsidize investment and wages in "have not" regions. Those regions often have tended to lag behind others, and to have suffered some depopulation. But in this model, they have not suffered enough depopulation.

Why these policies are so common is a question not addressed here. Certainly with the flexibility to make side–payments with which I endow it, the central government could make everyone better off by reversing the direction of subsidies.

Obviously a simple stylized model such as this one cannot be expected to explain all relevant behaviour. But the inconsistency between the federal policies prescribed here, and those actually carried out, is certainly a weakness.

Another impliction of this paper is that, absent central government intervention, there will be less strategic tax–setting in more finely divided countries. The optimality condition (12) can be used to show that as a nation of given population becomes more finely divided, the equilibrium regional tax rates converge. The only reason for inefficiency here is strategic behaviour, and the incentive to indulge in strategic behaviour disappears as a region's market power diminishes.

On the other hand, the federal government's ability to overcome the inefficiency depends on no single region being dominant. The model implies that the most effective federations will be those comprised of a fairly small number of equally–sized regions. Large numbers of regions reduce the importance of federation, and asymmetries may reduce the effectiveness of federal policy.

Of course, in this model there is no reason for regional governments to exist. Taken literally, the model suggests merger into a unitary state is a better idea even than federation.

The extension which I find most promising is to relax the assumption that all people born in a given region are the same. Many of the results obtained here continue to hold when resource owners are different people than workers, and when immobile resource owners control regional government. In particular, equation (12), the first–order condition for regional taxes, loses the term $-N_i$.

The conditions for efficiency of equilibrium are different, then, when resource owners are different from workers. That means the migration the federal government would like to induce may be different. However, the effects of any federal policy (grant or tax share) on the regions' wage tax rates are unchanged.

Perhaps a more complicated (and realistic) pattern of factor ownership among natives of a given region might lead to a better positive explanantion of the sort of regional policies in which federal governments actually indulge.

Appendix 1 : Proof That When There are no Federal Subsidies Nash Equilibrium is Efficient if and only if Regions are Identical, if and only if There is no Migration

Efficiency requires workers' marginal products to be the same in every region. If all s_j's are 0, then equation (2) implies that the equilibrium is efficient if and only if wage tax rates are the same in each region.

By regions being "identical", I mean that labour's marginal product in each region is the same, when there is no migration,

$$F_i'(N_i) = F_j'(N_j) \quad \text{all} \quad i, j$$

The theorem will be proved by showing identical regions imply an effient equilibrium, that an efficient equilibrium implies no migration, and then that no migration in equilibrium implies identical regions.

If regions are identical, then equation (12) shows that $t_i = g^*$ for each region i satisfies the first–order conditions for a Nash equilibrium. When each other region sets $t_j = g^*$, $t_i = g^*$ is also a global optimum for region i's citizens' incomes. Setting $t_i > g^*$ would result in $L_i < N_i$, so that $\partial W_i / \partial t_i < 0$, and setting $t_i < g^*$ would imply $\partial W_i / \partial t_i > 0$.

When regions are identical, could there be any other Nash equilibrium other than $t_i = g^*$ for all i? If all tax rates are identical, then $L_i = N_i$ for all i, so that equation (12) implies all tax rates must equal g^*. Suppose $t_i > t_j$ for some i and some j. Let i and j be the region's with the highest and lowest wage tax rates respectively. Then $L_i < N_i$ and $L_j > N_j$. Equation (12) then implies $t_i < g^* < t_j$, a contradiction.

So it has been shown that if regions are identical, then the unique Nash equilibrium involves equal wage tax rates in each region, and that this equilibrium is efficient.

Suppose the equilibrium is efficient. Then tax rates must be equal everywhere. Adding up equation (12) over all regions implies a weighted average of the regions' tax rates must equal g^*. Therefore, if all tax rates are equal, then they all equal g^*. Equation (12) then shows $L_i = N_i$ for all i.

Therefore, if the equilibrium is efficient, then there is no migration in equilibrium.

Suppose there is no migration in equilibrium. Then equation (12) implies $t_i = g^*$ for each i. The mobility condition (2) requires $F_i'(L_i) - t_i = F_j'(L_j) - t_j$, so that $F_i'(N_i) = F_i'(L_i) = F_j'(L_j) = F_j'(N_j)$. If there is no migration in equilibrium, then regions must be identical.

Appendix 2 : Details of Proof of Theorem 5

As shown in section 5, if the relative wage derivatives a_i can be taken as constant, then differentiation of equation (12) with respect to the wage tax rates and the location subsidies yields the system

$$\begin{pmatrix} (1-a_1^2) & -a_1a_2 & \cdot & \cdot & -a_1a_I \\ -a_1a_2 & (1-a_2^2) & \cdot & \cdot & -a_2a_I \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -a_1a_I & -a_2a_I & \cdot & \cdot & (1-a_I^2) \end{pmatrix} \begin{pmatrix} dt_1 \\ dt_2 \\ \cdot \\ \cdot \\ dt_I \end{pmatrix} = \begin{pmatrix} (1-a_1)(a_1+\zeta_1) \\ -a_1(a_2+\zeta_2) \\ \cdot \\ \cdot \\ -a_1(a_I+\zeta_I) \end{pmatrix} ds_1$$

The matrix M on the left side of the above equation has positive elements on the diagonal, negative elements off the diagonal, and column sums of $(1-a_i)(1+a_i) - a_i(1-a_i) = (1-a_i) > 0$. Therefore it is a dominant diagonal matrix, and must have a positive determinant (and a positive–valued inverse matrix).

Suppose first that region 1's citizens paid the entirety of the federal tax burden, so that $\zeta_1 = 1$ and all the other ζ_j's were 0. Then the column vector on the right–hand side of the above matrix equation would exactly equal the first column of the matrix on the left hand side. Then Cramer's Rule immediately implies

$$\frac{\partial t_1}{\partial s_1} = 1 \qquad \frac{\partial t_j}{\partial s_1} = 0 \quad j \neq 1 \qquad \text{when} \quad \zeta_1 = 1$$

Now if $\zeta_1 < 1$, the change in the column vector on the right–hand side of the above matrix equation, from when $\zeta_1 = 1$ is

$$(-(1-a_1)(1-\zeta_1), -a_1\zeta_2, \cdots, -a_1\zeta_I)' < 0$$

Since the matrix above has a positive–valued inverse, this implies the change in the vector of tax changes dt in moving the federal tax shares from $(1, 0, \cdots, 0)'$ to ζ must be negative. Therefore

$$\frac{\partial t_1}{\partial s_1} < 1 \qquad \frac{\partial t_j}{\partial s_1} < 0 \quad j \neq 1 \qquad \text{when} \quad \zeta_1 < 1$$

The matrix above can be written as $I - M$, where

$$M_{ij} = a_i a_j$$

Therefore, the inverse of the above matrix is

$$(I-M)^{-1} = I + M + M^2 + \cdots$$

Multiplying the matrix M by itself,

$$M \cdot M = (\sum_{i=1}^{I} a_i^2) M$$

Since each of the a_i's is less than 1, the sum of their squares is less than 1, which then implies

$$\sum_{i=1}^{\infty} M^i = \sum_{i=0}^{\infty} [\sum_{k=1}^{I} a_k^2]^i M = \frac{M}{m}$$

where

$$m = 1 - \sum_{i=1}^{I} a_i^2$$

That result means

$$(I - M)^{-1} = I + \frac{M}{m}$$

and the original system of derivatives can be solved as

$$\begin{pmatrix} dt_1 \\ dt_2 \\ \cdot \\ \cdot \\ dt_I \end{pmatrix} = \begin{pmatrix} (1-a_1)(a_1+\zeta_1) \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix} ds_1 + \frac{1}{m} M \begin{pmatrix} (1-a_1)(a_1+\zeta_1) \\ -a_1(a_2+\zeta_2) \\ \cdot \\ \cdot \\ -a_1(a_I+\zeta_I) \end{pmatrix} ds_1$$

so that

$$\frac{\partial t_1}{\partial s_1} = (1-a_1)(a_1+\zeta_1) + \frac{a_1^2}{m}(a_1+\zeta_1) - \frac{a_1^2}{m}[\sum_{k=1}^{I} a_k(a_k+\zeta_k)]$$

and

$$\frac{\partial t_j}{\partial s_1} = -a_1(a_j+\zeta_j) - \frac{a_1 a_j}{m}[\sum_{k=1}^{I} a_k(a_k+\zeta_k)] + \frac{a_1 a_j}{m}(a_1+\zeta_1)$$

It is straightforward to verify that when $\zeta_1 = 1$ and the other ζ_j's are 0, that $\partial t_1/\partial s_1 = 1$ and $\partial t_j/\partial s_1 = 0$ for $j \neq 1$. Moreover a decrease in ζ_1, with all other ζ_j's increasing or remaining constant must decrease all the $\partial t_i/\partial s_1$'s.

If each of the a_i's is less than or equal to 0.5, then concavity of m in the a_i's implies

$$m > 0.5$$

Then the rightmost term in the expression for $\partial t_1/\partial s_1$ cannot exceed a_1^2 in absolute value, so that

$$\frac{\partial t_1}{\partial s_1} > 0 \qquad \text{when} \quad a_i \leq \frac{1}{2} \quad i = 1, 2, \cdots, I$$

To examine the effect on mobility consider the difference in the net fiscal reward from moving from another region, say region 2, to region 1. This net reward is $s_1 - t_1 - s_2 + t_1$. From the above expressions,

$$1 - \frac{\partial t_1}{\partial s_1} + \frac{\partial t_2}{\partial s_1} = (1-a_1)(1-\zeta_1)[1 + \frac{a_1(a_1-a_2)}{m}] + \frac{a_1(a_1-a_2)}{m}\sum_{j=2}^{I} a_j\zeta_j - a_1\zeta_2$$

If $a_1 \geq a - 2$ then the only negative term in the above expression is the last one. Since the first term is at least as large as $(1-a_1)(1-\zeta_1)$, which muct exceed $a_1\zeta_2$ when $a_1 < 0.5$, the change in net fiscal reward must be positive when $0.5 \geq a_1 \geq a_2$.

Suppose then that $a_1 < a_2$. If all the relative wage responses a_i are less than or equal to 0.5, then $m \geq 0.5$. Since $\zeta_2 \leq (1-\zeta_1)$, and

$$\sum_{j=2}^{I} a_j\zeta_j < \sum_{j=2}^{I} \zeta_j = 1 - \zeta_1$$

then, when $a_2 > a_1$, the change in the difference in net fiscal reward must be at least

$$(1-\zeta_1)[(1-a_1)(1+2(a_1-a_2))] + 2a_1(a_1-a_2) - a_1]$$

Let

$$\phi(a_1) = (1-a_1)[1 + 2a_1(a_1-a_2)] + a_1(a_1-a_2) - a_1$$

Then

$$\phi'(a_1) = 8a_1 - 4a_2 + 4a_1a_2 - 6a_1^2 - 2$$

and

$$\phi''(a_1) = 8 - 4a_2 - 12a_1$$

Over the relevant range ($0 \leq a_1 \leq a_2 \leq 0.5$), $\phi''(a_1) < 0$. At $a_1 = 0$, $\phi'(a_1) = -4a_2 - 2 < 0$, so that $\phi(a_1)$ is decreasing over the relevant range. At the maximum of the range, $a_1 = a_2$,

$$\phi(a_2) = 1 - 2a_2 \geq 0$$

so that

$$\phi(a_1) \geq 0 \qquad \text{all} \quad 0 \leq a_1 \leq a_2 \leq 0.5$$

which implies that even when $a_2 > a_1$, the change in the difference in net fiscal reward must be positive.

Thus it has been shown that an increase in s_1 must increase $s_1 - t_1 - t_j$ for any other region j. This increase can occur only if there has been net immigration into region 1. (If there had been emigration from region 1, then the gross wage there would have increased. Then the gross wage would had to have increased in every other region, for residents to remain indifferent among regions. And the gross wage cannot rise in all regions if the total number of residents in all the regions is constant.)

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