The Advantages of Regional Disparities

The increased mobility of people among regions of a country, and among countries, has given rise to considerable concern about how this mobility might exacerbate regional disparities. If one country has a higher quality of productive infrastructure than another, the presence of increasing returns, or of positive externalities in production, may imply free migration will depopulate less–developed countries.

This paper concentrates not on the social costs associated with these regional disparities, but on their economic benefits. If increasing returns are important, and if geography matters, then there will be economic advantages to the concentration of economic activity in one location.

Here I present a simple model in which these economic advantages are emphasized. In this model, the efficiency of concentrating investment in one location follows almost immediately from the assumptions. The main issues addressed are how strategic considerations affect countries' investment policies.

The sort of strategic consideration which motivates the paper follows from the way in which the gains from concentration of investment might be allocated. Suppose that the country in which investment was concentrated reaped all the economic benefits of that concentration. Could it then be the case that another country would also invest heavily, leading to a wasteful duplication, because its citizens did not get the reward for providng cheap labour for the first country's industry?

In this paper, it turns out that such "over–investment" is not the chief problem. The main problem is "under–investment" by the country in which investment is to be concentrated. As might be expected, the nature of the negotiation between the countries will affect the efficiency of the policies they choose. Even though it leads to a sort of "hold–up" problem, allowing the countries to bargain over the terms of a treaty may be more efficient than a more "laissez–faire" procedure, in which no payments between countries are negotiated. Although bargaining for side payments between the countries tends to produce a more efficient allocation of investment, it tends to lead to a worse outcome for the country being depopulated.

The model presented here is a static model. Two countries decide on their investment plan, then decide whether to form a federation, then production takes place. In this model, what distinguishes a federation from two autonomous countries is the mobility of labour. Autonomy means no mobility at all between countries. Federation means free mobility between the constituent countries.

What is crucial here is that the investment plans are made prior to the federation decision. It is this timing which cause strategic considerations to bias the choice of investment plan. This artificial, multi–stage, one–period time frame is meant to telescope in a relatively tractable way an environment in which federation takes place over time ; the reductions in mobility barriers within the European Union have occurred over real time, and the constituent countries have continued to make their own policy decisions as the nature of the federation has evolved.

Three different ways of forming a federation are considered. First, I consider a federation in which each country is concerned only with the maximization of the total value of the federation itself. This would occur if there were some exogenously–specified sharing rule for this value. It serves mostly as a benchmark, since strategic considerations will not bias investment choices under such rules. Second, I consider a federation in which the allocation of income within the federation is determined by bargaining, bargaining which takes place after investment decisions have been made in each of the countries. Here, of course, an analogue to the hold–up problem may emerge, as countries' investment policies are determined by how their share of the income is affected, as well as the total income of the federation. Third, I consider a federation which really has no constitution at all. In this third case, federation simply means the free movement of people between countries.

In each of the three cases, the procedures for forming a federation (or deciding to remain autonomous) define a game played at the investment–choosing stage. In each case, the payoffs to each of the two countries will be determined by the two levels of investment. I assume that these choices are made simultaneously, and look at the Nash equilibrium to the game.

The paper is organized as follows. The first section presents the basic model. The second section examines the first way of forming the federation : I refer to this case as the "coordination game" since the two countries receive the same payoffs. The "bargaining game", the second of the three ways of forming a federation, is discussed in the following section, and the final way of forming a federation, the "laissez faire game" is discussed next. The concluding section discusses how the model might be extended.

1. The Model

As mentioned above, there are three stages in the model. First, two countries decide simultaneously on how much to invest in infrastructure. Second, the countries then decide on whether to federate. Third, investment and production take place.

The only agents are the two countries themselves. The payoff a country is assumed to maximize is the total income of its citizens. "Citizens" here may be different from residents ; in the most efficient outcome all of the citizens of one country go to work in the other country.

The number of countries is "large", so that the decisions of any two of them — in particular, their decision whether to form a federation — has no impact on world variables. The relative prices of goods are taken as given, and as unaffected by the decision of any two countries to federate.

If a country is autonomous, its workers cannot move to any other country. But they are freely mobile among the industries within the country. The labour supply of each worker is assumed exogenous. Workers can take part in three activities : producing goods in the "low–tech" sector ; producing goods in the "high–tech" sector, and producing investment.

Low–tech goods are produced by workers alone, without any capital. High–tech goods are produced by workers using the capital produced by workers in the investment sector. The investment good is a public intermediate good, produced directly by the government (which conscripts the necessary labour from the work force). The low–tech good is produced under constant returns to scale. The high–tech good is produced under increasing returns to scale.

In particular, the quantity of the high–tech good produced in a country is

$$(L^h)^\alpha K$$

if L^h workers work in high–tech production, and if the total investment is K. The exponent α is between 0 and 1, so that there are non–increasing returns in labour alone.

Units are chosen so that one worker produces one unit of the low–tech good. The price of the low–tech good is taken as numeraire, and p denotes the (exogenous) price of the high–tech good. Investment goods are produced from labour alone ; it takes k workers to produce one unit of the investment good.

Denote by $F(L,K)$ the maximum value of output in an autonomous country which chooses a level of investment K, and which has a labour force of size L. If L^h workers are allocated to the high–tech sector, then there will be $L - kK - L^h$ workers available for low–tech production, the value of the country's output will be

$$p(L^h)^\alpha K + L - kK - L^h$$

$F(K,L)$ is the maximum of this expression with respect to L^h, subject to the constraints that

$$0 \leq L^h \leq L - kK$$

The value of the marginal product of workers in the low–tech sector is 1 : by the choice of the low–tech good as numeraire, and the convention that each worker produces one unit of the low–tech good. Therefore, not surprisingly, the best allocation of workers is to allocate all workers to the high–tech sector if the value of their marginal product there exceeds 1, and to allocate them between sectors so as to equalize the value of their marginal product otherwise. Therefore, the optimal allocation of labour to the high–tech sector, $L^h(L,K)$ is the minimum of $L - kK$ and the solution to

$$\alpha p(L^h)^{\alpha-1}K = 1 \tag{1}$$

If the country is diversified, so that $L^h(L,K) < L$, then L^h is independent of L, and increasing in K. If the country is specialized, so that $L^h = L - kK$, then L^h is increasing in L and decreasing in K.

The country will be diversified if and only if

$$\alpha p(L - kK)^{\alpha-1}K < 1$$

The left side of the above expression is decreasing in K. Therefore there is some cut–off level of investment $\tilde{K}(L)$ such that the country will choose diversification if and only if $K < \tilde{K}(L)$. This $\tilde{K}(L)$ is strictly increasing in L. Solving equation (1), in this case

$$F(L,K) = B(pK)^\beta + L - kK \qquad \text{if} \quad K < \tilde{K}(L) \tag{2}$$

where

$$\beta \equiv \frac{1}{1-\alpha} > 1$$

$$B = \alpha^{\alpha\beta} - \alpha^{\beta}$$

If the country is specialized, then

$$F(L, K) = p(L - kK)^{\alpha} K \qquad \text{if} \quad K > \tilde{K}(L) \tag{3}$$

and $\tilde{K}(L)$ is defined by

$$(\alpha p)^{\beta} [\tilde{K}(L)]^{\beta} = L - k\tilde{K}(L) \tag{4}$$

The fact that $\beta > 1$, implied by the increasing returns to scale, means that the value of output $F(L, K)$ is a convex function of the investment level K if the country is to be diversified. However, if the country is to be specialized, then

$$F_K(L, K) = p(L - kK)^{\alpha-1}[L - (\alpha + 1)kK] \qquad \text{if} \qquad K > \tilde{K}(L) \tag{5}$$

which means that $F(L, K)$ is a quasi–concave function of K alone when there is specialization.

It then follows that there are two possible levels of investment which might maximize the value of the country's output. If $K = 0$, then the country certainly will be diversified (where by "diversified" I mean : "producing some of the low–tech good"). Differentiation of equation (2) shows that $F_K(L, K) < 0$ at $K = 0$, so that doing no investment at all constitutes a local maximum to the country's problem. There can be no interior solution to the country's maximization in which $0 < K \leq \tilde{K}(L)$, from the fact that $F(L, K)$ is convex as a function of K over this range. And there is a unique local maximum to the problem in the region of specialization, namely

$$K^s(L) = \frac{1}{(1 + \alpha)k} L \tag{6}$$

which equation (5) shows is the unique solution to $F_K(L, K) = 0$ in the region of specialization.

The value of the country's output if it chooses not to invest is simply L, since then all labour will be allocated to low–tech production. The maximum value of output if it chooses a positive level of investment will be

$$F(L, K^s(L)) = \alpha^{\alpha}(1 + \alpha)^{-(1+\alpha)} \frac{p}{k} L^{1+\alpha} \tag{7}$$

Notice that if the payoff from specialization exceeds the payoff from diversification, then

$$p(L - K^s(L))^{\alpha} K^s(L) > L$$

so that

$$\alpha p(L - K^s(L))^{\alpha-1} K > 1$$

and $K^s(L) > \tilde{K}(L)$.

Equation (7) shows that the payoff from specialization is a convex function of the total labour force L, while the payoff from diversification is simply equal to L. Therefore, in equilibrium, it is the countries with the largest populations which will invest in infra–structure, and which will specialize. Assuming that the relative price of the high–tech good goes to zero as total world production of the low–tech good goes to zero, then some countries must be producing some of the low–tech good. That means that world equilibrium is characterized by some threshold level $\tilde{L}$ of labour force size. All countries with a population of less than $\tilde{L}$ undertake no investment in infra–structure, and produce only the low–tech good. All countries with $L > \tilde{L}$ specialize in high–tech production, and invest $K^s(L)$.

Also, for future reference, the marginal payoff to investment in the initial stage when the country is diversified can be derived, from equation (3), as

$$F_K(L, K) = \beta B p^\beta K^{\alpha\beta} - k \qquad \text{if} \quad K < \tilde{K}(L) \tag{8}$$

2. The Coordination Game

If two countries federate, then workers are assumed to become freely mobile between the countries. However, capital remains immobile. Further, the federation occurs after the countries have made their commitments to investment plans. Therefore, a federation's allocation decision is where to use the $L_1 + L_2 - kK_1 - kK_2$ available in the two countries. These workers can be used in high–tech production in either country, or in low–tech production.

Let $G(L_1, L_2, K_1, K_2)$ denote the maximum value of the federation's output, the maximum value of

$$p[L_1^h]^\alpha K_1 + p[L_2^h]^\alpha K_2 + L_1 + L_2 - L_1^h - L_2^h - kK_1 - kK_2 \tag{9}$$

with respect to L_1^h and L_2^h, subject to the constraints that the L_i^h's be non–negative, and that $L_1 + L_2 - L_1^h - L_2^h - kK_1 - kK_2$ be non–negative.

First of all, note that federation cannot reduce the value of total output. The federation always has the option of choosing the same L_1^h and L_2^h as its constituent countries would choose if autonomous.

Given the investment plans (K_1, K_2) that have already been made, federation will increase the value of output if and only if the marginal product of labour in the two autonomous countries' high–tech sectors differs.

LEMMA 1 : $G(L_1, L_2, K_1, K_2) > F(L_1, K_1) + F(L_2, K_2)$ if and only if

i at least one country would be specialized if autonomous

and

ii $\alpha(L_1 - kK_1)^{\alpha-1}K_1 \neq \alpha(L_2 - kK_2)^{\alpha-1}K_2$

PROOF : As long as the marginal product of labour differs in the two countries' high–tech sectors, then a slight reallocation of workers from the low–productivity country to the high–productivity country will increase the value of the federation's output.

But if labour's marginal product is the same in each country's high–tech sector, no such profitable reallocation can occur, given that labour's marginal product in high–tech production declines strictly with the quanity of labour employed.

The value of labour's marginal product in two autonomous countries will be equal if both countries are diversified — in which case the value of the marginal product equals 1 — or if both are specialized and $\alpha(L_i - kK_i)^{\alpha-1}K_i$ is the same in each country. otherwise the marginal products will differ.

In a federation, workers should be allocated between the high–tech industries in the two constituent countries so as to equalize their marginal products. More specifically, take the solution (L_1^h, L_2^h) to the following pair of equations :

$$L_1^h + L_2^h = L_1 + L_2 - kK_1 - kK_2$$

$$\alpha p(L_1^h)^{\alpha-1}K_1 = \alpha p(L_2^h)^{\alpha-1}K_2 \tag{10}$$

The solution to these equations is

$$L_1^h = \frac{K_1^\beta}{K_1^\beta + K_2^\beta}[L_1 + L_2 - k(K_1 + K_2)] \tag{11}$$

$$L_2^h = \frac{K_2^\beta}{K_1^\beta + K_2^\beta}[L_1 + L_2 - k(K_1 + K_2)] \tag{12}$$

If the solution involves both sides of equation (10) being greater than or equal to 1, then this solution is the optimal allocation of labour to the two countries' high–tech industries, and no labour should be allocated to low–tech production. If the solution involves both sides of equation (10) being less than 1, then the optimal allocation is to use

$$[\alpha p K_i]^\beta$$

workers in high–tech production in country i, and use the remaining $L_1 + L_2 - kK_1 - kK_2 - [\alpha p]^\beta[(K_1)^\beta + (K_2)^\beta]$ workers in low–tech production.

In the first case, in which the federation specializes in high–tech production, equations (11) and (12) (and a little manipulation) imply

$$G(L_1, L_2, K_1, K_2) = p[K_1^\beta + K_2^\beta]^{1-\alpha}[L_1 + L_2 - k(K_1 + K_2)]^\alpha \tag{13}$$

and in the second case, when the federation produces some of the low–tech good

$$G(L_1, L_2, K_1, K_2) = Bp^\beta[K_1^\beta + K_2^\beta] + L_1 + L_2 - k(K_1 + K_2) \tag{14}$$

The boundary between these two regimes, specialization in high–tech production, and diversification, is the set of investment plans (K_1, K_2) which satisfy the following equation

$$(\alpha p)^\beta (K_1^\beta + K_2^\beta) = L_1 + L_2 - k(K_1 + K_2) \qquad (15)$$

Equation (15) defines a downward–sloping curve in K_1–K_2 space. The region of diversification, the area below this curve, is a convex set. The boundary starts at $(0, \tilde{K}(L_1 + L_2))$, and goes down to $(\tilde{K}(L_1 + L_2), 0)$. Also, from equation (4), the point $(\tilde{K}(L_1), \tilde{K}(L_2))$ is exactly on this boundary. Figure 1 illustrates the regions of diversification and specialization for a federation. Because $(\tilde{K}(L_1), \tilde{K}(L_2))$ is on this boundary, it follows that a federation will be specialized if both constituent countries would have been specialized when autonomous, and the federation will be diversified if both constituent countries would have been diversified when autonomous.

In this figure, there are 6 possibles regions. Each region is labelled by : what happens in country 1 if it is autonomous, what happens in country 2 if it is autonomous, and what happens in the federation of the two countries (so that sdD, for example, is the region in which country 1 is specialized, country 2 is diversified, and the federation is diversified.) The point on the graph where all 6 regions intersect is $(\tilde{K}(L_1), \tilde{K}(L_2))$.

If the federation were able to pick the investment levels in its two constituent countries so as to maximize the value of its output $G(K_1, K_2, L_1, L_2)$, then it would never choose to invest in both countries. This result is hardly surprising. To see the result, suppose that the federation allocates workers optimally, for some given investment pattern (K_1, K_2) in which both K_1 and K_2 are positive. The total output from the federation's high–tech production is $(L_1^h)^\alpha K_1 + (L_2^h)^\alpha K_2$. Now consider changing the investment plan to $(0, K_1 + K_2)$. One feasible allocation of workers with this new plan is to allocate $L_1^h + L_2^h$ workers to high–tech production in country 2, leaving low–tech production unchanged. The federation's total high–tech production is now $(L_1^h + L_2^h)^\alpha (K_1 + K_2)$ which certainly must be higher than before.

It also does not matter to the federation whether the investment is concentrated in one country or the other ; free mobility of labour means workers will go to where the investment is.

Therefore, if the federation were to choose an investment to maximize the value of its output, it suffices to look at investment plans in which $K_1 = 0$. If $K_1 = 0$, then

$$G(L_1, L_2, K_1, K_2) = F(L_1 + L_2, K_2) \qquad \text{if} \quad K_1 = 0 \qquad (16)$$

The following lemma then follows immediately :

LEMMA 2 : The investment plan which maximizes the value $G(L_1, L_2, K_1, K_2)$ of the federation's output is $K_1 = K_2 = 0$ if $L_1 + L_2 \leq \tilde{L}$, and is $K_1 = 0$, $K_2 = K^s(L_1 + L_2)$ (or $K_1 = K^s(L_1 + L_2)$, $K_2 = 0$) if $L_1 + L_2 > \tilde{L}$.

Implicit in the above formulation is the assumption that workers can be imported from one country to the other in order to produce the latter country's investment. That is, while countries commit to their investment plans before federation, they may rely on imported workers to undertake it. This possibility of importing labour is relevant only if

$$L_i < \frac{L_1 + L_2}{(1+\alpha)}$$

which must be the case for the smaller country — and may be the case for both countries.

If workers could move only for high–tech production, but not to undertake investment, then it would matter where investment was undertaken. The added feasability constraints would be

$$kK_i \leq L_i \quad I = 1, 2$$

If $L_1 \geq L_2$, the optimum would then involve $K_1 = K^s(L_1 + L_2)$ and $K_2 = 0$ if

$$L_1 \geq \frac{L_1 + L_2}{(1+\alpha)}$$

(but not the reverse investment plan). If the above inequality did not hold (for example, if the two countries had nearly the same population), then the optimum would involve $K_1 = \frac{L_1}{k}$. If α were sufficiently small, the optimum would then also involve positive investment in the second country as well. So changing the assumption about mobility of investment, so that investment must be undertaken using only domestic workers, will change the nature of the optimum. However, henceforth it will be assumed that investment can be undertaken using migrant workers, so that it need not be the case that $kK_i \leq L_i$.

Lemma 2 looks at the overall optimal policy for a federation. It will also prove useful to consider what level of investment in one country is best for the federation, given the level of investment in the other constituent country. That is, given that country 1 has committed to an investment level of K_1, what level of investment K_2 maximizes the value $G(L_1, L_2, K_1, K_2)$ of the federation's output?

If the federation is diversified, then the net marginal contribution of investment in country 2 is

$$\frac{\partial G}{\partial K_2} = p(L_2^h)^\alpha - k \tag{17}$$

which must be an increasing function of K_2, since $L_2^h = (\alpha p K_2)^\beta$ if the federation is diversified. As in the case of an autonomous country, the value of output is a convex function of either country's investment if the federation is diversified.

If the federation is specialized, then the net marginal contribution of investment in country 2 is

$$\frac{\partial G}{\partial K_2} = p(L_2^h)^{\alpha-1}[L_2^h - \alpha k K_2] \tag{18}$$

As investment gets large enough — specifically as K_2 approaches $L_1 + L_2 - kK_1$, expression (18) must become negative. Now if the expression is zero, this need not represent a local maximum for $G(L_1, L_2, K_1, K_2)$ with respect to K_2 ; differentiation of the above expression shows that it need not be decreasing in K_2 when it equals zero.

However, for a given (L_1, L_2, K_1), there can be at most one local maximum for $G(L_1, L_2, K_1, K_2)$. $\blacksquare$ That is, if $\partial G/\partial K_2 = 0$ for some K_2, and the second–order condition for a maximum with respect to K_2 holds, then that second –order condition must hold at any larger K_2 for which the first–order condition holds. Denote this local maximum as

$$K_2^*(L_1, L_2, K_1)$$

Then for any given K_1, there are three possibilities for the "best response" for country 2, which maximizes the federation's output given (L_1, L_2, K_1). First, $G(L_1, L_2, K_1, K_2)$ may be everywhere decreasing in K_2, so that the optimal investment response is $K_2 = 0$. Second, there may be a locally optimal $K_2^*(L_1, L_2, K_1) > 0$, which yields a lower value of federal output than the globally maximal $K_2 = 0$. Finally, $K_2^*(L_1, L_2, K_1)$ may be globally optimal.

Next, note that for any give level of K_2, that L_2^h is non–increasing in K_1. If the federation is diversified, then L_2^h is independent of K_1. If it is specialized, then equation (12) shows that L_2^h decreases strictly with K_1. From equation (17), then, $\partial G/\partial K_2$ is independent of K_1 when the federation is diversified. When the federation is specialized, equation (18) shows that $\partial G/\partial K_2$ decreases with K_1, holding constant K_2.

This effect of K_1 on the marginal return to investment in country 2 has several implications. First, if G is a monotonically decreasing function of K_2 for some given K_1, then it will be monotonically decreasing for all larger K_1'. Second, where there is a local maximum $K_2^*(L_1, L_2, K_1)$, it is a decreasing function of K_1. Third, the relative advantage of this local maximum over not investing at all, $G(L_1, L_2, K_1, K_2^*[L_1, L_2, K_1]) - G(L_1, L_2, K_1, 0)$ is a strictly decreasing function of K_1.

That means that there is a downward sloping reaction function in the coordination game, defining the values of K_2 which maximize $G(L_1, L_2, K_1, K_2)$ given K_1. This reaction function jumps discontinuously down to 0 at some threshold level of investment in country 1.

Next, consider the value $K_2^*(L_1, L_2, K_1)$ which (locally) maximizes the federation's output, given K_1. How does this value compare with the value $K^s(L_1)$ which (locally) maximizes the value of the autonomous country 2. $K_2^*(L_1, L_2, K_1)$ will exceed $K^s(L_2)$ if and only if L_2^h is higher in the federation with (L_1, L_2, K_1, K_2) than in the autonomous country with (L_2, K_2) — at $K_2 = K^s(L_2)$. That in turn will be the case if federation causes an inflow of workers from high–tech production in country 1 to high–tech production in country 2. Workers will move to the country where there marginal productivity is highest. Country 2 is specialized when it is autonomous and when $K_2 = K^s(L)$. The marginal productivity of labour in its high–tech industry is

$$\alpha p(L_2 - kK^s(L_2))^{\alpha-1}K^s(L)$$

The marginal productivity of labour in the other country will be lower under independence if

$$(L_2 - kK^s(L))^{\alpha-1}K^s(L) > (L_1 - kK_1)^{\alpha-1}K_1$$

Using the definition (6) of $K^s(L_2)$, it then follows that $K_2^*(L_1, L_2, K_1) > K^s(L_2)$ if and only if

$$(\alpha)^{\alpha-1}(1+\alpha)^{-\alpha}(k)^{-1}L_2^{\alpha} > (L_1 - kK_1)^{\alpha-1}K_1 \quad (19)$$

If $L_1 + L_2 > \tilde{L}$, then the efficient outcome is for one country to invest $K^s(L_1 + L_2)$, and the other to invest nothing. It does not matter which country does the investing. This efficient outcome is a Nash equilibrium. First of all, if $K_1 = 0$,then country 2's best response is to pick the investment which maximizes $G(L_1, L_2, 0, K_2)$, namely $K^s(L_1 + L_2)$. But

$$G(L_1, L_2, K, K^s(L_1 + L_2)) < G(L_1, L_2, 0, K + K^s(L_1 + L_2)) < G(L_1, L_2, 0, K^s(L_1 + L_2))$$

for any $K > 0$, where the first inequality results from it always being best to concentrate investment in one country, and the second from the optimality of $K^s(L_1+L_2)$. Hence $K_1 = 0$ is a best reaction to $K_2 = K^s(L_1 + L_2)$.

There may, however, be an inefficient Nash equilibrium to this game : a coordination failure. Note that any equilibrium in which both countries undertake positive levels of investment must involve specialization in the federation. Under specialization, the marginal cost of investment, namely the marginal productivity of labour in the high–tech industry, must be the same in each country. Therefore, if K_1 and K_2 are each best responses for countries 1 and 2, then the marginal productivity of investment must be the same in each country, or $L_1^h = L_2^h$. Equations (11) and (12) then imply that $K_1 = K_2$ in any Nash equilibrium in which K_1 and K_2 are both positive. The common level of high–tech employment must be

$$L^h = \frac{1}{2}(L_1 + L_2 - kK) \quad (20)$$

where K is the common level of investment.

If this pair of investment choices $K_1 = K_2 = K$ is a Nash equilibrium, then (from equation (18)), it must be the case that $L^h = \alpha kK$, so that

$$K = \frac{1}{2}K^s(L_1 + L_2) \quad (21)$$

The only possible inefficient Nash equilibrium to this investment coordination game involves the efficient total, except the investment divided equally between the two countries.

Let $\bar{L}$ denote the average population of the countries. Since $K^s(L)$ is proportional to L, in this candidate equilibrium, each country chooses $K = K^s(\bar{L})$. When the two countries choose the same level of investment, each gets half the work force allocated to it, so that the overall value of the federation's output in this candidate equilibrium is

$$2F(\bar{L}, K^s(\bar{L}))$$

The best possible deviation for either country from this candidate Nash equilibrium is to an investment level of 0. In this case, the overall value of the federation's output is

$$F(2\bar{L}, K^s(\bar{L})$$

Now if $K^s(\bar{L}) > \tilde{K}(2\bar{L})$, then a country with $2\bar{L}$ workers and an investment level $K^s(\bar{L})$ would be specialized. Then the value of its output would be

$$\alpha p(2\bar{L} - K^s(\bar{L}))^\alpha K^s(\bar{L})$$

which, from the definition of $K^s(L)$, exceeds $F(\bar{L}, K^s(\bar{L})$ by a factor

$$(2\alpha + 1)^\alpha$$

This number will exceed 2 if $\alpha > 0.5$.

Since the payoff from deviation is at least $p(2\bar{L} - K^s(\bar{L}))^\alpha K^s(\bar{L})$, and will be strictly higher if diversification is optimal for a country with $L = 2\bar{L}$ and $K = K^s(\bar{L})$, deviation to $K_i = 0$ must be better than the candidate Nash equilibrium, if $\alpha > 0.5$.

LEMMA 3 : There is no coordination problem if $\alpha > 0.5$.

PROOF : The above argument shows that the candidate for a Nash equilibrium in which K_1 and K_2 were both positive would not actually be a Nash equilibrium, provided that $\alpha > 0.5$. But a Nash equilibrium in which both countries invest at a positive level can arise only if the federation is specialized. And then the first–order conditions for each country are satisfied only if $K_1 = K_2 = K$ and $L_1^h = L_2^h = L^h$, and if equations (19) and (20) are satisfied. Therefore, there is only one possible Nash equilibrium in which both countries invest a positive amount — and unilateral deviation from that "possible" Nash equilibrium will always be optimal if $\alpha > 0.5$.

Any Nash equilibrium in which one country invests nothing must be efficient. If $K_1 = 0$, then country 2's reaction involves maximization of $G(L_1, L_2, 0, K_2)$ with respect to K_2, which results in (K_1, K_2) maximizing $G(L_1, L_2, K_1, K_2)$.

Moreover, any efficient (K_1, K_2) pair will be a Nash equilibrium, regardless of the relation between $L_1 + L_2$ and $\tilde{L}$.

Therefore, if $\alpha > 0.5$, the set of Nash equilibria in pure strategies to the coordination game is exactly the set of efficient investment plans.

The condition $\alpha \leq 0.5$ is necessary but not sufficient for the existence of an inefficient Nash equilibrium to the coordination game. The condition certainly guarantees that it is not profitable to deviate to an investment level of 0 if the federation stays specialized when $K_1 = 0$, $K^2 = K^s(\bar{L})$. But in such a case it may be optimal to be diversified. That is, when $\alpha < 0.5$, it is possible that

$$F(2\bar{L}, K^s(\bar{L})) > 2F(\bar{L}, K^s(\bar{L})) > p[2\bar{L} - K^s(\bar{L})]^\alpha K^s(\bar{L})$$

This possibility occurs, for example, when $p = 0.2$, $\alpha = 0.45$, $\bar{L} = 20$, $k = 0.3$.

3. The Bargaining Game

The game in which countries commit to investment, and then federate, and in which they have a common goal, is a bit contrived. But it does provide something of a benchmark to a different game, in which there is less commonality of interest.

In such a game, one can assess whether strategic considerations distort countries' investment decisions, rather than coordination problems. Implicit in the previous game was the notion that income earned in a federation was divided among the constituent countries equally, or at least in a fashion that is unaffected by the countries' investment decisions. Such a rule for allocation of income may have peculiar implications. One of the features of the inefficient coordination equilibrium, should it exist, is that there are no gains from federation. For example, if the earnings of the federation are distributed equally per person to the $L_1 + L_2$ residents of the federation, then the aggregate income of the larger of the two countries would actually be lower under federation than under independence. (This result applies only in the inefficient coordination equilibrium in which there is investment in both countries.)

In the "bargaining game" the distribution of a federation's income to its countries is something which is determined endogenously. For simplicity, I assume that the payoffs to the countries are determined by the Nash bargaining solution. That means that the payoff to country 1 is

$$\pi_1(L_1, L_2, K_1, K_2) = \frac{1}{2}[F(L_1, K_1) + G(L_1, L_2, K_1, K_2) - F(L_2, K_2)] \tag{22}$$

with an analogous expression for country 2. Here I have assumed the bargaining takes place after the countries have committed to their investment decisions. Then the disagreement point is the pair of payoffs the countries would get should they remain autonomous.

The following proposition follows almost immediately :

PROPOSITION 1 : There is no efficient Nash equilibrium to the bargaining game.

PROOF : The efficient investment plan involves one country (say country 1) choosing $K_1 = 0$, and the other country choosing $K_2 = K^s(L_1 + L_2)$. By definition

$$\frac{\partial}{\partial K_2} G(L_1, L_2, 0, K^s(L_1 + L_2)) = 0$$

But $K^s(L_1 + L_2) > K^s(L_2)$, and $F(L_2, K_2)$ is strictly concave in K_2 when region 2 is specialized. Therefore

$$\frac{\partial}{\partial K_2} F(L_2, K^s(L_1 + L_2)) < 0$$

so that equation (22) shows that country 2's payoff is strictly decreasing in K_2 at the efficient investment plan.

In this game, then, the problem tends to be one of under–investment. Increases in investment beyond $K^s(L_i)$ weaken country i's bargaining position by making the disagreement point worse for it.

But the fact that country 2's best reaction to $K_1 = 0$ is some K_2 between $K^s(L_2)$ and $K^s(L_1 + L_2)$ does not ensure that there exists a Nash equilibrium in which one country invests nothing and the other country invests too little. Country 1's best reaction to K_2 may be a positive level of investment.

There must be at least one Nash equilibrium in pure strategies to this game.

LEMMA 4 : There is at least one Nash equilibrium in pure strategies to the bargaining game.

PROOF : It has already been established that $\partial G/\partial K_2$ is strictly decreasing in K_1 when the federation is specialized. It is independent of K_1 when the federation is diversified.

Therefore $\partial \pi_2/\partial K_2$ is strictly decreasing in K_1 if the federation is specialized, and otherwise is independent of K_1.

It then follows that the set of best responses of country 2 to K_1 is non–increasing in K_1 : if K_2 is among the best responses of country 2 to K_1, and $K_2' > K_2$ is among the best responses of country 2 to $K_1' > K_1$, then K_2 must also be a best response to K_1' and K_2' a best response to K_1.

Let $f_2(K_1)$ be the maximum of country 2's best responses to country 1. Since $f_2(K_1)$ is a non–increasing function of K_1, it consists of continuous, non–decreasing segments, and a finite number of jump discontinuities. The analogous $f_1(K_2)$ has the same properties.

If K_2 is sufficiently large, country 2's best response must be 0. If country 1's best response to 0 is in this range of values for which country 2's best response is 0, we have a Nash equilibrium. Otherwise the graph of f_1 starts out (on the K_2–axis) above the graph of f_2. Similarly, if K_1 is large enough, then country 2's best response is 0. If there is no Nash equilibrium for which $K_2 = 0$, then the graph of f_1 winds up (along the K_1–axis) below the graph of f_2.

So it remains to show only that there must be a Nash equilibrium if the graph of f_1 starts out above the graph of f_2 and winds up below it. In this case, consider the graphs obtained by "filling in" the jump discontinuities in f_1 and f_2. Since these new curves are closed, then the "filled in" graph of f_1 must intersect the "filled in" graph of f_2, crossing it from above. But any intersection of these "filled in" graphs cannot occur in the "filled in" jump discontinuities, if the graph of f_1 crosses from above. Therefore, there must be at least one intersection of the "filled in" graphs which also represents points which are on the original f_i's and thus constitute a Nash equilibrium in pure strategies.

Two types of (pure strategy) Nash equilibrium are possible in this game. One type involves one country investing nothing. In such an equilibrium, if country 1 invests nothing, country 2 will invest some K_2 which is between $K^s(L_2)$ and $K^s(L_1 + L_2)$. In fact, its optimal reaction to $K_1 = 0$ must be less than $K^s(\frac{L_1}{2} + L_2)$. This pair of investment levels will constitute a Nash equilibrium if country 1's best reaction to this level of K_2 is indeed $K_1 = 0$.

If not, and if the "reverse" configuration ($K^s(L_1) < K_1 < K^s(L_1 + \frac{L_2}{2})$, $K_2 = 0$) is also not an equilibrium, then there will be at least one Nash equilibrium in which both countries invest positive amounts.

Either possibility can arise. If $L_1 = 11$, $L_2 = 9$, $k = 1$ and $\alpha = 0.7$, then there is a Nash equilibrium with country 1 not investing, providing the price of the high–tech good is low enough. If $p = 1$, then $K_1 = 8.52$ and $K_2 = 0$ is a Nash equilibrium. Country 2 would rather set $K_2 = 0$ than deviate to its best positive–valued response to $K_1 = 8.52$, which here is 2.33.

A higher price for the high–tech good does not affect one country's best response to the other country's investment of 0 : when both the autonomous country 1 and the federation are specialized, then π_1 is proportional to p. However, it raises the relative payoff to country 2 from deviating from 0 : $K_2 = 0$ means country 2 will be diversified if autonomous, and it will be specialized if autonomous at its best possible positive deviation. If $p = 5$, then country 2's best response to $K_1 = 8.52$ is now $K_2 = 2.33$.

In this case, there must be a Nash equilibrium in which K_1 and K_2 are both positive : here $K_1 = 7.94$ and $K_2 = 2.78$.

The example suggests that a higher price for the high–tech good makes it less likely that the Nash equilibrium involves one country investing nothing.

It also will be the case that greater asymmetries between countries make it more likely there is a Nash equilibrium in which the smaller country does not invest, and make such a Nash equilibrium more efficient.

Let

$$K_1^R(L_1, L_2, K_2)$$

denote the "restricted" best response of country 1 to country 2 in the bargaining game : that is, the positive value for K_1 which is a local maximum to country 1's payoff (and which may or may not yield a higher payoff than $K_1 = 0$). Then

LEMMA 5 : Holding $L_1 + L_2$ constant, an increase in L_1 will increase $K_1^R(L_1, L_2, 0)$, and will decrease the payoff to country 2 from deviating from $K_2 = 0$ to $K_2 = K_2^R(L_1, L_2, K_2^R(L_1, L_2, 0))$.

PROOF : Differentiation of equation (3) shows that

$$\frac{\partial^2 F}{\partial K \partial L} = \alpha p[L - kK]^{\alpha-2}[L - \alpha kK] > 0 \tag{23}$$

and equation (13) shows that $G(L_1, L_2, K_1, K_2)$ does not change when L_1 and L_2 change so as to leave $L_1 + L_2$ constant.

$K_1^R(L_1, L_2, 0)$ is the value of K_1 which maximizes $F(L_1, K_1) + G(L_1, L_2, K_1, 0)$. When $K_2 = 0$, $G(L_1, L_2, K_1, 0) = F(L_1 + L_2, K_1)$, which is a strictly concave function of K_1 if the federation is specialized. Therefore equation (23) implies that an increase in L_1, and an equal decrease in L_2 must increase $K_1^R(L_1, L_2, 0)$, since the change raises $F_K(L_1, K_1)$ and leaves constant $G(L_1, L_2, K_1, 0)$.

The payoff to country 2 from deviation from $K_2 = 0$ to any $K_2 > 0$ is

$$F(L_2, K_2) + G(L_1, L_2, K_1^R(L_1, L_2, 0), K_2) - F(L_2, 0) - G(L_1, L_2, K_1^R(L_1, L_2, 0), 0)$$

When K_2 is country 2's best positive reaction to K_1, then the envelope theorem implies the change in this deviation [from the increase in L_1 and the decrease in L_2] is

$$\frac{\partial}{\partial K_1}[G(L_1, L_2, K_1^R(L_1, L_2, 0), K_2) - G(L_1, L_2, K_1^R(L_1, L_2, 0), 0]d[K_1^R(L_1, L_2, 0)]$$
$$+\frac{\partial}{\partial L_2}[F(L_2, K_2) - F(L_2, 0)]dL_2$$

Since

$$\frac{\partial^2 G}{\partial K_1 \partial K_2} \leq 0$$

and $dK_1^R > 0$, the first term in the above expression must be non–positive. Equation (22) shows that the second term must be negative.

Therefore, deviation becomes less attractive for country 2 as L_1 increases and L_2 decreases.

This lemma shows that increased asymmetry between countries makes federation more attractive in two ways. First, the under–investment by the larger country is reduced. Second, the temptation to over–invest by the small country is reduced.

In general there may be many Nash equilibria. It certainly is possible that there may be two Nash equilibria in which one country under–invests. Lemma 5 does imply that if $L_1 \geq L_2$ and there is a Nash equilibrium in which $K_1 = 0$, then there also must be a Nash equilibrium in which $K_2 = 0$. It also implies that the value of the federation's output is no lower in the equilibrium in which $K_2 = 0$.

In the coordination game any equilibrium with positive investment by both countries had to be symmetric. That is no longer the case when countries' payoffs are determined by the Nash bargaining solution. In fact, $K_1 = K_2 > 0$ can be a Nash equilibrium to this game only if $L_1 = L_2$.

Further limits can be placed on the equilibrium investment levels in an equilibrium in which both those levels are positive. The ratio of investment levels cannot be "intermediate", in the following sense.

LEMMA 6 : If K_1 and K_2 are both positive in some Nash equilibrium to the bargaining game, then $L_1 \geq L_2$ implies either

$$K_1 < K_2$$

or

$$\frac{K_1}{K_2} > \frac{L_1}{L_2}$$

PROOF : Suppose that $L_1 > L_2$.

Equation (3) shows that $F(L, K)$ is homogeneous of degree $\alpha + 1$ in K and L. Differentiation of that equation also shows that $F_K(K, L)$ decreases with K/L.

Therefore, from the concavity of $F(K, L)$ when countries are specialized,

$$F_K(L_1, K_1) > F_K(L_2, K_2) \quad \text{if and only if} \quad \frac{K_1}{L_1} < \frac{K_2}{L_2} \tag{24}$$

From equations (11) and (12), in a federation

$$\frac{L_1^h}{L_2^h} = (\frac{K_1}{K_2})^\beta$$

implying that

$$\frac{K_1}{L_1^h} > \frac{K_2}{L_2^h} \quad \text{if} \quad \text{and} \quad \text{only} \quad \text{if} \quad K_2 > K_1 \tag{25}$$

Equation (18) can be written

$$\frac{\partial G}{\partial K_i} = p(L_i^h)^\alpha [1 - \alpha k \frac{K_i}{L_i^h}] \tag{26}$$

Next, note that at any Nash equilibrium in which $K_i > K_j > 0$, it must be the case that

$$\frac{\partial G}{\partial K_i} > 0 > \frac{\partial G}{\partial K_j}$$

Why? The derivative of the value of output with respect to investment is proportional to

$$1 - \alpha k \frac{K_i}{L_i^h}$$

whether the country is autonomous or specialized. If $K_i > K_j$, then L_i^h would be higher under federation than under specialization, meaning that $\partial G / \partial K_i < 0$ would imply $F_K(L_i, K_i) < 0$, contradicting the first–order conditions for a maximum.

Suppose now that

$$K_1 > K_2 > 0$$

The above discussion implies that

$$\frac{\partial G}{\partial K_1} > 0 > \frac{\partial G}{\partial K_2}$$

For (K_1, K_2) to constitute a Nash equilibrium, then,

$$F_K(L_2, K_2) > 0 > F_K(L_1, K_1)$$

which requires

$$\frac{K_1}{L_1} > \frac{K_2}{L_2}$$

Analogously, if

$$\frac{K_1}{L_1} < \frac{K_2}{L_2}$$

then

$$F_K(L_1, K_1) > F_K(L_2, K_2)$$

meaning

$$\frac{\partial G}{\partial K_2} > \frac{\partial G}{\partial K_1}$$

so that $K_2 > K_1$.

Differentiation of the first–order conditions for a Nash equilibrium in a neighbourhood of $L_1 = L_2$ shows that an increase in L_1 and an equal decrease in L_2 must increase K_2 and reduce K_1 so that

LEMMA 7 : If L_1 and L_2 are sufficiently close to each other in value, then $L_1 > L_2$ implies $K_1 > K_2$ in any Nash equilibrium to the bargaining game in which both investment levels are positive.

PROOF : Start with $L_1 = L_2$ and $K_1 = K_2$.

If L_1 increases by Δ and L_2 decreases by Δ, then differentiation of each country's first–order condition implies

$$\begin{pmatrix} G_{11} + F_{KK} & G_{12} \\ G_{12} & G_{11} + F_{KK} \end{pmatrix} \begin{pmatrix} dK_1 \\ dK_2 \end{pmatrix} = \begin{pmatrix} F_{KL} \\ -F_{KL} \end{pmatrix} \Delta$$

where G_i refers to the derivative of G with respect to K_i.

It has already been established that $G_{12} < 0$. Second–order conditions ensure that $G_1 1 + F_{KK} < 0$.

Differentiation of equation (18) shows that $G_{11} < G_{12}$ when $L_1 = L_2$, $K_1 = K_2$ and $G_1 = 0$.

Therefore the above matrix has an inverse with negative elements on the diagonal, and positive elements off the diagonal. The fact that $F_{KL} > 0$ then establishes the result.

It is perhaps a familiar theme that inefficient investment is induced by the bargaining which takes place after the countries have committed to their investment decisions. Efficiency requires that each country gain the full benefit at the margin from some change in its investment decision which increases the payoff to the federation. Typically, if the payoffs to investment in some game have this property, then the payoffs do not add up properly, to the value of the federation's output in this case.

4. The Laissez–Faire Game

Suppose now that no bargaining takes place between prospective members of a federation. Each country instead faces a choice of joining the federation or not. (As before, this choice is made after the countries' commitment to investment policies.) Further, there are no transfers within the federation, and no interference with the markets. If federation occurs, workers are free to move, and are paid the value of their marginal product.

Therefore, the total net income of residents of a country, should that country join a federation, is the profit earned (net of wages) by that country's high–tech sector, plus the wages earned by residents of the country. I assume that the government of each country cares only about this total income earned by all (original) residents of the country. Therefore, should country 1 join a federation with country 2, the payoff earned by the country is

$$\pi_1(L_1, L_2, K_1, K_2) \equiv p(L_1^h)^\alpha K_1 + w[L_1 - kK_1 - L_1^h] \tag{27}$$

where w is the wage in the federation, and where $L_1^h$is employment in the country's high–sector in the federation. The sum of the two countries' payoffs is exactly $G(L_1, L_2, K_1, K_2)$.

Under this "laissez–faire" payoff structure, each country would indeed choose to join a federation.

LEMMA 8 : In the laissez–faire game, $\pi_1(L_1, L_2, K_1, K_2) \geq F(L_1, L_2)$ for any (L_2, K_2)

PROOF : Consider the following function $H(L_1, K_1, L_1^h)$:

$$H(L_1, K_1, L_1^h) \equiv p(L_1^h)^\alpha L_1^h + W(K_1, L_1^h)[L_1 - kK_1 - L_1^h] \tag{28}$$

where $W(K_1, L_1^h)$ is the value of labour's marginal product in the high–tech sector given a labour force of L_1^h employed there, and an investment level of K_1.

For a given L_1 and K_1, when L_1^h is the level of high–tech employment resulting from autonomy, then

$$H(L_1, K_1, L_1^h) = F(L_1, K_1)$$

and when L_1^h is the level of employment resulting from federation, then

$$H(L_1, K_1, L_1^h) = G(L_1, L_2, K_1, K_2)$$

The derivative of H with respect to L_1^h is

$$\frac{\partial H}{\partial L_1^h} = \frac{\partial W}{\partial L_1^h}[L_1 - kK_1 - L_1^h] \tag{29}$$

The wage is a decreasing function of the quantity employed in the high–tech sector.

If the wage under autonomy is less in country 1 than in country 2, then federation will result in a reduction in L_1^h as labour moves from country 1 to country 2. But when labour moves from country 1 to country 2 $L_1 - kK_1 - L_1^h$ becomes positive. Therefore equation (29) shows the increase in L_1^h caused by federation must increase H, so that $\pi_1(L_1, L_2, K_1, K_2) > F(L_1, K_1)$.

If the wage under autonomy is greater in country 1 than in country 2, then federation will decrease L_1^h. In this case, the fall in L_1^h means that $L_1 - kK_1 - L_1^h$ becomes positive, so that the decrease in L_1^h increases $H(L_1, K_1, L_1^h)$.

Therefore, federation must yield a higher payoff in this game than autonomy — except if the two countries had the same wage under autonomy, in which case federation yields the same payoff.

Given that w is the value of the marginal product of labour in each country's high–tech sector (if the country has a high–tech sector), it follows from equation (27) that

$$\frac{\partial \pi_1}{\partial K_1} = \frac{\partial G}{\partial K_1} + \frac{\partial w}{\partial K_1}[L_1 - kK_1 - L_1^h] \tag{30}$$

If the federation is diversified, then $w = 1$ autonomous of K_1, and the country's marginal payoff from investment is the same as the federation's. But if the federation is specialized, then w is an increasing function of K_1. Therefore, the marginal payoff to investment in country 1 exceeds the marginal value of investment to the federation if (and only if) $L_1 - kK_1 - L_1^h > 0$. Under specialization, this inequality holds if and only if labour moves from country 1 to country 2 in a federation.

In particular, an efficient outcome cannot be sustained as a Nash equilibrium to this game. In such an outcome, one country (say country 1) does all the investing, and $K_1 = K^s(L_1 + L_2)$ if $L_1 + L_2 > \tilde{L}$. But the "terms–of–trade" effect of investment on the wage rate imply that π_1 is decreasing in K_1 at $K_1 = K^s(L_1 + L_2)$, $K_2 = 0$.

As under Nash bargaining, there are two types of Nash equilibrium possible in the laissez–faire game : in one type one country chooses not to invest, and in the other both choose to invest. As under Nash bargaining, if it is efficient for the federation to be specilized, then the federation will be specialized in any Nash equilibrium.

Consider now one particular (potential) Nash equilibrium, the Nash equilibrium arising when $L_1 > L_2$, and in which $K_2 = 0$ in equilibrium. Such an equlibrium may arise either in the bargaining game, or in this laissez–faire game. But, if the small country is very small, then the laissez–faire game's equilibrium gives rise to a lower value of output of the federation, and a lower payoff to the large country. However, the small country will be better off under laissez–faire.

PROPOSITION 2 : Let $L_1 > L_2$. Suppose that both the bargaining game and the laissez–faire game have Nash equilibria in which $K_2 = 0$. Then if L_2 is sufficiently small, K_1 is higher in the bargaining game, and π_1 is higher as well.

PROOF : Consider varying L_2, starting from $L_2 = 0$, in a federation in which $L_1 > \tilde{L}$ (so that the federation should be specialized). At $L_2 = 0$, both games have efficient equilibria, in which $K_1 = K^s(L_1 + L_2)$. The payoff to country 1 will be the same in both games when $L_2 = 0$, namely $F(L_1, K^s(L_1))$.

The derivative of country 1's payoff in the bargaining game, with respect to L_2, is

$$\frac{1}{2}[\frac{\partial G}{\partial L_2} - \frac{\partial F(L_2, 0)}{\partial L_2}] = \frac{1}{2}[w - 1]$$

where w is the value of labour's marginal product in a specialized federation. This derivative is strictly positive at $L_2 = 0$.

The derivative of country 1's payoff with respect to L_2 in the laissez–faire game is

$$-\frac{\partial w}{\partial L_2} L_2 = (1 - \alpha) w \frac{L_2}{L_1 + L + 2 - kK_1}$$

which equals 0 when $L_2 = 0$.

Therefore, if the small country is small enough, the payoff of the large country will be smaller in the laissez–faire game than in the Nash bargaining game.

In either game, the change in the equilibrium K_1 as L_2 changes equals

$$-\frac{\partial^2 \pi_1}{\partial L_2 \partial K_1} / \frac{\partial^2 \pi_1}{\partial (K_1)^2}$$

where π_1 is country 1's payoff in the game.

From the above expressions for $\partial \pi_1 / \partial L_2$,

$$\frac{\partial^2 \pi_1}{\partial L_2 \partial K_1} = \frac{1}{2} \frac{\partial w}{\partial K_1} > 0$$

in the bargaining game.

In the laissez–faire game,

$$\frac{\partial^2}{\partial L_2 \partial K_1} = (1-\alpha) \frac{L_2}{L_1 + L_2 - kK_1} [\frac{\partial w}{\partial K_1} + \frac{wk}{L_1 + L_2 - kK_1}]$$

This expression is non–negative, but equals 0 when $L_2 = 0$. Therefore there is a higher K_1 in the bargaining game.

The derivative of the small country's payoff with respect to L_2 is

$$\frac{1}{2}[w+1] + \frac{1}{2} \frac{\partial K_1}{\partial L_2} [\frac{\partial G}{\partial K_1} - \frac{\partial F(L_1, K_1)}{\partial K_1}]$$

This second term is 0 at $L_2 = 0$, so that the derivative equals the average of w and 1.

Under laissez–faire, the derivative of the small country's payoff with respect to L_2 is

$$w + \frac{\partial w}{\partial L_2} L_2$$

Again the second term disappears when $L_2 = 0$, so that the derivative under laissez–faire is $w > \frac{w+1}{2}$.

What has been described here as a federation in the laissez–faire game may better be thought of as two autonomous countries, who have chosen to allow migration. In fact, lemma 8 shows that each country will prefer allowing migration to cutting it off completely.

But the laissez–faire behaviour here also involves allowing labour markets to function competitively. A country which is receiving net immigration certainly has some incentives to undertake a more activist policy. If tax revenue can be distributed in lump–sum fashion to citizens of the country, the government has an incentive to tax wage income, and to redistribute the proceeds to citizens. If the Nash equilibrium involves $K_1 > 0$ and $K_2 = 0$, then a small wage tax has no effect on the allocation of workers. Since the country has committed to its investment plan, the wage tax cannot alter the number of workers involved in investment good production. All other workers in either country work in high–tech production in country 1, where they earn a wage of $w > 1$. A wage tax on employment in country 1 of $t < w - 1$ will not induce any worker to move ; their next–best opportunity is in low–tech production in country 2, earning a wage of 1.

Therefore, if lump–sum payments to citizens are possible, and if country 2 somehow were to stick with a laissez–faire policy, country 1 would wish to levy a wage tax of at least $w - 1$. It certainly would want the tax sufficiently high that some workers were induced to switch to low–tech production in country 2. On the other hand, if all workers earn a net wage of 1, country 2 may have an incentive to subsidize work in its own low–tech industry. The subsidy will raise the gross wage in country 1.

A Nash equilibrium to this tax–and–subsidy game would be a tax rate t in country 1 and a subsidy rate s in country 2 which satisfy

$$\frac{\partial \pi_1}{\partial t} = \frac{\partial w}{\partial t}[L_1 - kK_1 - L_1^h] + L_1 = 0 \tag{31}$$

$$\frac{\partial \pi_2}{\partial s} = -\frac{\partial w}{\partial t}[L_1 - kK - L_1^h] - L_2 + (w-1)\frac{\partial L_1^h}{\partial t} = 0 \tag{32}$$

where the allocation of labour to country 1's high–tech sector now obeys

$$\alpha p(L_1^h)^{\alpha-1}K_1 - t = 1 + s \tag{33}$$

Equations (31)–(33) now imply that in Nash equilibrium

$$(1-\alpha)(L_1 - L_2) = \frac{(w-1)}{w}L_1^h \tag{34}$$

If countries cannot commit themselves to avoid this sort of strategic tax setting after the federation, then equations (33) and (34) characterize the outcome from the investment choices K_1 and $K_2 = 0$. This outcome will not involve specialization ; it will involve inefficient under–migration, as low–tech production persists in country 2.

Of course, the Nash equilibrium to the earlier investment–setting stage of the game now becomes more complicated, as the payoffs depend on the post–federation tax–subsidy game (which is more complicated than equations (31)–(34) if both countries have positive investment levels).

5. Extensions

The main conclusions from this model are : concentration of infrastructure invvestment in one country may be efficient ; countries' investment incentives are distorted if they make investment decisions prior to deciding to federate ; over–investment by small countries is less of a problem than under-investment by a big country ; efficiency may be enhanced by allowing countries to bargain over transfers before setting up a federation.

Of course the extreme nature of the first conclusion — that efficiency implies the entire population of one country should work in another country — is an artifact of the simplicity of the model. It would certainly disappear if there were more than one high–tech good, as in Dixt–Stiglitz type models (provided the infrastructure used for one high–tech good was useless in the production of any other high–tech good). A more realistic specification of mobility costs would also modify this conclusion.

The third conclusion, that small countries tend not to over–invest, follows from the fact that their optimum is a corner solution. The distortions in their marginal payoffs induced by strategic considerations may not alter their optimal decision. The possibility of such a corner solution is relatively robust ; it follows from the convexity of the technology.

The efficiency of one country invested nothing does depend on the assumption that migrant labour can be used for investment, even though the investment commitment has to be made prior to the federation decision. As mentioned in section 2, without this possibility it may be efficient to undertake investment in both countries : the big country should use all its labour for investment, and the small country should use some of its labour. (This case would arise if the two countries were similar in population, and if α were small.) In such a case, over–investment in the small country in the bargaining game would be the norm.

Of course, it seems rather artifical that the commitment to invest takes place before the investment itself is undertaken. A more natural — but less tractable — extension of the model would be to have investment take place before production, in other words to use a multi–period model.

Extending the model in this way has several interesting implications. Suppose, for example, that the infrastructure K_i available in country i in some period depended on the investment done in all previous periods. Would countries which had invested heavily in the past be the ones which continued to invest in infrastructure? The increasing returns to scale imply that the payoff to a given level of investment is indeed higher in a country with a high initial level of infrastructure. But so is the opportunity cost of investment ; the wage will be higher in high–infrastructure countries.

In a world of autonomous countries, with no possibility of federation, it may be the case that the opportunity cost effect dominates. The equilibrium will involve "leapfrogging" whereby countries which invest little in the first period will invest a lot in the second period.

In fact, investing little in the first period means investing nothing in the first period, given the convexity. If initially there is no infrastructure at all, then countries in the first period should either invest nothing at all, or devote their entire labour force to investment in infrastructure (which will only be productive in the next period). leapfrogging involves some of the countries which invested nothing in the first period then choosing to invest in subsequent periods.

In such a world, there will be strategic over–investment if the federation decision is made after several periods. After federation, it certainly is best to concentrate investment in one country. This termination of all future investment after federation makes investment prior to federation less profitable for the other country. So efficiency may involve concentration of all pre–federation investment in one country. However, if leapfrogging is the equilibrium outcome, positive investment in this country may have strategic advanatages, by increasing the disagreement payoff.

Moreover, a multi–period framework would allow for evolving federalism. If post-federation investment were concentrated in one country, the relative bargaining power of that country would increase over time. If parties to a federation could not commit to a system of transfers at the time of federation, this change in bargaining power could have severe effects. The small country would either refuse to join the federation (anticipating future exploitation as the constitution is renegotiated), or demand the right to (inefficiently) continue to invest after federation in order to maintain its bargaining power.

Another extension is to allow more than two countries to federate. Given the role geography plays (implicitly) here, allowing countries to bid for partners seems a bit far–fetched. But if two countries federate, then they may subsequently choose to add another contiguous country to the federation. This extension seems to describe the European experience. It also seems to demand a more complicated game–theoretic structure.

It is assumed here that each country is "small". It might be more realistic to allow countries some market power over goods prices. This introduces another bias into the investment decision, whether or not there is federation. The incentive to invest is reduced for a country which has invested a lot : further investment in infrastructure reduces the relative price of the high–tech good which it exports. Conversely, countries with a low level of investment would have an incentive to over–invest because of the effect on the terms of trade.

Countries are assumed to be homogeneous here. Of course this assumes away many important issues involving the distribution of the gains from investment, and involving decision–making among heterogeneous agents. But the decision of the government to maximize the total income of its citizens is particularly contentious when migration is important. Here I assumed that a country's decision makers regard a dollar earned by one of the country's citizens as being equally valuable, wherever the citizen earns it. But when people migrate to work in countries with higher levels of infrastructure, they may migrate permanently. Politicians in their country of origin may not care for their welfare so much if they cease to vote in that country. It may be that concern over depopulation by politicians reflects a bias in the political structure : migration of agents from one country to another may make the migrants better off, but this increase in their well–being may not increase the payoff to decision makers in either country. Perhaps a multi–period model, in which decision makers maximize the welfare of residents of their country at the beginning of the period, can deal with some of these issues.