*Q*1. What should the tax rate τ on the return to saving be, if the government must raise some given revenue R by taxation of labour income at the rate t, and of the return to saving at the rate τ, if the economy consists of identical people with the following characteristics? People live for two periods, and work only in the first period. Their preferences can be represented by the utility function

$$U(L, c_1, c_2) = \alpha \ln(1 - L) + \beta \ln c_1 + \gamma \ln c_2$$

where L is work done in period 1, c_i is consumption in period i, and $\alpha + \beta + \gamma = 1$. The wage rate w is exogenous, as is the return r to saving.

Since savings tax revenue is collected in the second period, the government revenue requirement is that the **present value** of revenue from the two taxes (wage tax revenue collected in period 1 and savings tax revenue collected in period 2) equal R. The government can borrow and lend at the same rate r as individuals.

*A*1. First, what is the indirect utility for a consumer whose direct untility function is $U(\bar{x}) = \sum_{i=1}^{n} a_i \ln x_i$, with $\sum_{i=1}^{n} a_i = 1$? With these Cobb–Douglas preferences, the person's Marshallian demand for good i would be

$$x_i^M(\bar{p}, Y) = \frac{a_i Y}{p_i} \tag{1-1}$$

where Y is her (exogenous) income, and p_i the price of good i.

That means that her indirect utility function, $V(\bar{p}, Y) \equiv U(\bar{x}^M(\bar{p}, Y))$ is

$$V(\bar{p}, Y) = \ln Y + \sum_{i=1}^{n} a_i \ln a_i - \sum_{i=1}^{n} a_i \ln p_i \tag{1-2}$$

where I have used the fact that $\ln AB = \ln A + \ln B$.

In the question, there are 3 goods : present leisure, present consumption, and future consumption. If

$$\omega \equiv w(1 - t)$$

$$\rho \equiv r(1 - \tau)$$

denote the net–of–tax wage rate and rate of return to saving respectively, then, using present consumption as the numéraire, her "exogenous" income is

$$Y \equiv \omega \tag{1-3}$$

since ω is what she would earn, net of tax, if she worked full time. Here the price of leisure is also ω. The price of present consumption is 1, since it is the numéraire, and the price of future consumption is $1/(1 + \rho)$. Therefore, using $(1 - 2)$, her indirect utility is

$$V(\omega, 1, \frac{1}{1+\rho}; \omega) = \ln \omega + \alpha \ln \alpha + \beta \ln \beta + \gamma \ln \gamma - \alpha \ln \omega + \gamma \ln(1 + \rho) \tag{1-4}$$

where I have used the facts that $\ln 1 = 0$, and that $\ln A/B = \ln A - \ln B$.

From $(1-4)$,

$$\frac{\partial V}{\partial \omega} = \frac{1-\alpha}{\omega} \qquad (1-5)$$

$$\frac{\partial V}{\partial \rho} = \frac{\gamma}{1+\rho} \qquad (1-6)$$

Now the government's budget constraint is

$$twL + \frac{1}{1+r}\tau r S = R \qquad (1-7)$$

where S is the person's saving : this constraint says that the present value of taxes raised must equal R, and that the government uses the market interest r to discount second–period tax payments.

From the person's Marshallian demand function $(1-1)$,

$$1 - L = \alpha \qquad (1-8)$$

$$c_1 = \beta\omega \qquad (1-9)$$

which then imply that

$$L = 1 - \alpha \qquad (1-10)$$

and

$$S = \omega L - c_1 = \omega(1-\alpha-\beta) = \gamma\omega \qquad (1-11)$$

Hence the government's problem is to pick t and τ so as to maximize the indirect utility of a representative agent, subject to the budget constraint $(1-7)$, which now can be written

$$tw(1-\alpha) + w(1-t)\tau\frac{\gamma r}{1+r} \qquad (1-12) = R$$

From the definition of ω and ρ,

$$\frac{\partial \omega}{\partial t} = -w \qquad (1-13)$$

$$\frac{\partial \rho}{\partial \tau} = -r \qquad (1-14)$$

Using $(1-5)$ and $(1-6)$, maximizing utility subject to the government budget constraint $(1-12)$ implies first–order conditions

$$\frac{\partial \mathcal{L}}{\partial t} = -\frac{1-\alpha}{1-t} + \lambda[(1-\alpha)w - w\tau\frac{\gamma r}{1+r}] = 0 \qquad (1-15)$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -\gamma\frac{r}{1+r(1-\tau)} + \lambda[w(1-t)\frac{\gamma r}{1+r}] = 0 \qquad (1-16)$$

where $\mathcal{L}$ denotes the Lagrangean, and λ the Lagrange multiplier on the budget constraint $(1-12)$.

Equation $(1-16)$ implies that

$$\lambda w = \frac{1+r}{(1-t)(1+r(1-\tau))} \tag{1-17}$$

Substituting $(1-17)$ into $(1-15)$,

$$(1-\alpha) - \tau\frac{\gamma r}{1+r} = (1-\gamma)\frac{1+r}{1+r(1-t)} \tag{1-18}$$

or

$$[-\frac{\gamma r}{1+r}]\tau = [(1-\alpha)\frac{r}{1+r(1-\tau)}]\tau \tag{1-19}$$

The coefficient of τ on the left side of $(1-19)$ is strictly negative ; the coefficient of τ on the right side is strictly positive. So $(1-19)$ can hold only if

$$\tau = 0$$

With these preferences, the optimal tax is an expenditure tax, in which all revenue is raised from a tax on labour income.

This result can be obtained in other ways. For example, one of the results of optimal commodity taxation is that goods c_1 and c_2 should be taxed at the same rate if preferences are separable in $1-L$, and if the "sub–utility" function for c_1 and c_2 is homothetic. Cobb–Douglas preferences must satisfy these conditions. So it is optimal to tax consumption in each period at the same rate. This can be achieved only if the return to saving is not taxed ; otherwise the relative price of c_1 and c_2 would be changed by the tax system.

*Q*2. How should a firm finance its investment if it faces a corporate income tax rate of 40 percent, if the personal income tax rate applicable on interest income is 50 percent, and the effective personal income tax rate on the return to equity is 20 percent?

*A*2. If the personal income tax rate on interest income is geater than the corporate income tax rate, then using retained earnings to finance investment is preferable to borrowing. Using retained earnings to finance investment is always preferable to issuing equity to finance investment, if the return to equity is taxed at a positive rate.

So if the firm has enough retained earnings available, then investment should be financed from retained earnings.

If the firm does not have sufficient retained earnings, then it must choose between debt and equity. In this case, debt is preferable to equity if and only if

$$1+r(1-\tau) > (1-\tau_d)(1+r(1-t)) \tag{2-1}$$

where t is the corporate tax rate, τ the personal tax rate on interest income, and τ_d the personal income tax rate on the return to equity, and where r is the required rate of return to investors.

Plugging in the values given in the question for the tax rates, condition $(2-1)$ becomes

$$1 + (0.5)r\tau > (0.8)(1 + (0.6)r) \tag{2-2}$$

But condition $(2-2)$ simplifies to

$$1 + (0.5)r > 0.8 + (0.48)r \tag{2-3}$$

which must hold, if the required rate of return r is non–negative.

Therefore, borrowing will be preferable to new equity issue here. The firm should use retained earnings to finance investment, and if the retained earnings are not sufficient, then it should borrow the rest.

*Q*3. What is the cost of capital on an asset with a true (exponential) depreciation rate of 20 percent, if the corporate income tax rate were 50 percent, the required rate of return of investors were 10 percent, the depreciation rate allowed on the asset for tax purposes were 40 percent, and the firm financed half of its investment by borrowing? (None of the costs of investment can be written off immediately.)

*A*3. A formula here : suppose that an asset's return declines at the constant rate γ, that the corporate income tax rate were t, and that the firm was allowed to claim depreciation against tax, using the declining balance mthod, with an (exponential) rate δ for taxable depreciation. Suppose as well that the firm could deduct a fraction ϕ of its capital expenditures immediately from its taxable income, using the declining balance method for the remaining fraction $1-\phi$ of the asset's cost.

Then if p is the price of the firm's output, and x is the marginal product — per dollar invested – of a new machine, then spending an addiitonal dollar on investment will yield the firm a net increase of

$$e^{-\gamma u}p(1-t)x + t(1-\phi)\delta e^{-\delta u} \tag{3-1}$$

in net income, when the asset is u years old.

If the firm discounts its net–of–tax earnings at a rate ρ, then the present vaue of this net return to investment is

$$\frac{(1-t)px}{\gamma+\rho} + \frac{t(1-\phi)\delta}{\rho+\delta} \tag{3-2}$$

It will invest up to the point where this present value of the net reurn equals the net cost of a dollar of investment, which is $1-\phi t$ if a fraction ϕ of the asset's cost can be deducted immediately from taxable income, implying that the firm should invest up to the point at which

$$\frac{(1-t)px}{\gamma+\rho} + \frac{t(1-\phi)\delta}{\rho+\delta} = 1 - \phi t \tag{3-3}$$

The cost of capital R is the discount rate which could be applied to the **before tax** returns to investment, which would give the same investment criterion as $(3-3)$. In other words, R solves

$$px = R + \gamma \qquad (3-4)$$

exactly when $(3-3)$ holds. Combining $(3-3)$ and $(3-4)$, implies that

$$(R+\gamma)\frac{1-t}{\rho+\gamma} = (1-\phi t) - (1-\phi)\frac{t\delta}{\rho+\delta} \qquad (3-5)$$

In this question, $\gamma = 0.2$, $t = 0.5$, $\delta = 0.4$ and $\phi = 0$. Also the firm is assumed to finance half of its investment by borrowing, and half by equity issue. Since only borrowing costs are tax deductible, the effective rate of return is r on equity, and $r(1-t)$ on debt. So the firm's discount rate should be a weighted average of r and $(1-t)r$. With exactly half the investment financed by borrowing, then

$$\rho = (0.5)r + (0.5)r(1-t) = r(1-\frac{t}{2}) \qquad (3-6)$$

With $r = 0.1$, this implies that

$$\rho = 0.075$$

Plugging these values into $(3-5)$ implies that

$$(R+0.2)\frac{0.5}{0.075+0.2} = 1 - \frac{(0.5)(0.4)}{0.075+0.4} \qquad (3-7)$$

or

$$R = \frac{0.275}{0.5}(1-\frac{0.2}{0.475}) - 0.2 \qquad (3-8)$$

or

$$R = \frac{9}{76} \approx 0.118$$

Eeven though a relatively rapid write–off of assets is allowed, here the fact that only half the firm's finance costs are tax deductible implies that the cost of capital is higher than the required rate of return : the corporate tax discourages investment here.

*Q*4. How would the price of houses vary with the Canadian income tax rate, in a simple world (no uncertainty, no inflation, only one tax bracket), if the interest rate in Canada were exogenously determined on world markets?

*A*4. If h is the annual rent on a house — net of any maintenance costs and other expensses — then asset market equilibrium requires that

$$h = r(1-t)H \qquad (4-1)$$

where H is the price of the house, r the interest rate, and t the income tax rate.

Why? $r(1-t)$ is the net–of–tax return on financial assets, if the return to financial assets is taxed at a rate t. In equilibrium, individuals must be indifferent between investing in owner–occupied housing and investing in other assets.

Equation $(4-1)$ relates the price of a house to the annual rent. To complete the model, note that the quantity supplied of housing must equal the quantity demanded. The quantity supplied depends on the price builders get for a new house, H, whereas the quantity demanded depends on the annual rental rate h (which is the user cost of living in owner–occupied housing). So

$$D(h) = S(H) \qquad (4-2)$$

Equations $(4-1)$ and $(4-2)$ together determine h and H as functions of the tax rate (and of the interest rate). They can be combined as

$$S(H) - D(r[1-t]H) = 0 \qquad (4-3)$$

implying that

$$\frac{\partial H}{\partial t} = \frac{1}{1-t}\frac{\epsilon_D}{\epsilon_D + \epsilon_S} \qquad (4-4)$$

where ϵ_D and ϵ_S are the (absolute values of the) demand and supply elasticities of housing :

$$\epsilon_D \equiv -D'(h)\frac{h}{D(h)}$$

$$\epsilon_S \equiv S'(H)\frac{H}{S(H)}$$

The higher is the elasticity of housing demand, relative to the supply elasticity, the more tax preferences are capitalized into the price of housing.

*Q*5. Suppose an asset goes up each year at the same rate, that the inflation rate is constant (and less than the rate at which the asset's nominal price rises), and that the individual sells the asset after T years. She must pay income tax on a fraction α of the nominal realized capital gain when she sells the asset.

Derive an expression for the "equivalent" tax rate on real, accrued capital gains. That is, find a tax rate τ which, if it were levied on real capital gains as they accrued, left the owner of the asset exactly as well off as under the "actual" tax (in which a fraction α of nominal gains are taxed on realization).

How does this effective tax rate vary with the rate of inflation?

*A*5. Let g be the annual rate of increase in the real value of the asset, and i the annual rate of increase in the price level.

Then if the asset were worth \$1 originally, its price would be

$$P_T \equiv (1+g)^T(1+i)^T \qquad (5-1)$$

after T years. If the person sold the asset after T years, and paid income taxes at the rate t on a fraction α of the nominal capital gains, then she would have

$$P_T - \alpha t(P_T - 1) = (1 - \alpha t)(1 + g)^T(1 + i)^T + \alpha t \qquad (5-2)$$

after she paid her taxes on the realized capital gain.

On the other hand, if real capital gains were taxed at the rate τ as they accrued, then the real value of the asset would grown only at the rate $1 + g(1 - \tau)$ per year. If the person had to pay taxes on the real capital gains as they accrued, then her wealth would be

$$(1 + g(1 - \tau))^T(1 + i)^T \qquad (5-3)$$

after T years.

The "equivalent" tax rate τ on real accruals is the rate which makes expression $(5 - 3)$ the same as expression $(5 - 2)$. That is

$$(1 + g(1 - \tau))^T(1 + i)^T = (1 - \alpha t)(1 + g)^T(1 + i)^T + \alpha t \qquad (5-4)$$

or

$$1 + g(1 - \tau) = (1 - \alpha t)^{1/T}(1 + g) + (\alpha t)^{1/T}(1 + i)^{-T} \qquad (5-5)$$

Equation $(5-5)$ shows that the effective tax rate must **increase** with the inflation rate $i :$ increasing i will decrease the right side of $(5-5)$, so that τ must increase as well to make the left side increase.

This result shows that the favourable tax treatment of capital gains (realization rather than accrual, low inclusion rate) is not really a "correction" for taxing nominal, rather than real, capital gains. It says that, if we were to keep the effective rate constant, then the inclusion rate α would actually **increase** with the inflation rate i, holding constant the real rate of growth g in the asset's value.