

do all 5 questions : all count equally

1. Find all the Pareto optimal allocations in the following 2-person, 2-good economy, in which good X is a pure private good and good Z is a pure public good.

The production possibility frontier for the economy has the equation

$$2X + Z = 270$$

where X and Z are the aggregate quantities produced of the pure private good and of the pure public good, respectively.

There are 2 people : person 1's preferences can be represented by the utility function

$$U^1(x_1, z_1) = 2 \ln x_1 + \ln z_1$$

and person 2's by the utility function

$$U^2(x_2, z_2) = \ln x_2 + 2 \ln z_2$$

where x_h and z_h are person h 's consumption of the pure private good and pure public good respectively.

2. Suppose that there are only two people in a country. Person 1's preferences can be represented by the utility function

$$U^1(x_1, z_1) = x_1 + Az_1 - \frac{1}{2}\alpha(z_1)^2$$

and person 2's by the utility function

$$U^2(x_2, z_2) = x_2 + Az_2 - \frac{1}{2}\beta(z_2)^2$$

where x_h and z_h are person h 's consumption of a pure private good and of a pure public good respectively.

The government knows A , but asks individuals 1 and 2 each to report a value a (for person 1) and b (for person 2) to measure the slope parameter in their demand curves for the public good.

The government promises to choose a public good level

$$Z^* = \frac{2A - c}{a + b}$$

and to tax person h

$$\frac{cZ^*}{2} + T_h$$

where

$$T_1 = (Z^* - Z_1) \left[\frac{b}{2} (Z^* + Z_1) - \frac{2A - c}{2} \right]$$

$$T_2 = (Z^* - Z_2) \left[\frac{a}{2} (Z^* + Z_2) - \frac{2A - c}{2} \right]$$

where the variables Z_1 and Z_2 are defined by

$$Z_1 = \frac{2A - c}{2b}$$

$$Z_2 = \frac{2A - c}{2a}$$

If the government commits to these rules for public good provision and taxation, what values a and b will the people report? Explain your answer.

3. How much of a deficit or surplus will the government run, if it uses the public good provision and taxation rules defined in question #2 above?

4. A country consists of 3 million people, 1 million of each type. Each person's preferences can be represented by the utility function

$$U^h = x_h + \ln z_h$$

where x_h is her consumption of a pure private good and z_h her consumption of a pure public good.

The public good must be financed by a proportional income tax, at the rate τ .

Each person's income is exogenous, but different types have different income. Type 1 people have income of 5, type 2 people have income of 10, and type 3 people have income of 30.

The cost of 1 unit of the public good is 3000000.

If people vote over the tax rate τ (with all tax revenue being used for public good provision), which tax rate will be chosen?

5. If it were also possible to make lump-sum transfers among people, find an allocation which is Pareto-preferred to the one chosen in the voting in question #4 above.