

do all 5 questions : all count equally

1. Find all the Pareto optimal allocations in the following 2–person, 2–input, 2–good economy. The quantity F produced of food is

$$F = k_F + l_F$$

where k_F and l_F are the quantities of machinery and labour used in food production.

The quantity C of clothing produced is

$$C = \sqrt{k_C l_C}$$

where k_C and l_C are the quantities of machinery and labour used in clothing production.

The total quantities of inputs available in the economy are 120 units of machinery and 240 units of labour. Both of these quantities are fixed.

Person 1's preferences can be represented by the utility function

$$u^1(F_1, C_1) = \ln F_1 + 2 \ln C_1$$

and person 2's by the utility function

$$u^2(F_2, C_2) = 2 \ln F_2 + \ln C_2$$

where (F_1, C_1) is person 1's consumption bundle of food and clothing, and (F_2, C_2) is person 2's.

2. There are 2 people in the economy. Each person's preferences can be represented by the same utility function :

$$u = c - \frac{1}{2}n^2$$

where c is the person's consumption, and n is the quantity of labour that she provides.

The consumption good is produced by labour alone. Person 1 can produce 1 unit of the consumption good per unit of her labour provided, and person 2 can produce 2 units the consumption good per unit of labour provided. (So the only difference between people is that person 2's labour is twice as productive.)

What are all the Pareto optimal allocations in this economy?

3. Suppose now that, in the economy described in question #2 above, there were not 2 people, but 2 million, 1 million of each type. Otherwise the technology and preferences are exactly as described in question #2.

If the government's only possible tax were a proportional wage income tax — at the same rate on each person's wage income — what tax rate would type-1 people prefer, if the government returns its tax revenue to the 2 million people, dividing it equally among them?

4. If the initial prices of goods 1, 2 and 3 are all 1, and if a person's expenditure function can be written

$$E(p_1, p_2, p_3, u) = p_1 u + 3(p_1 p_2 p_3)^{1/3}$$

then what is the excess burden of a tax system which taxes good 2 at a proportional rate of τ_2 , good 3 at a proportional rate of τ_3 , and does not tax good 1 (so that the tax-inclusive prices of the three goods are 1, $1 + \tau_2$ and $1 + \tau_3$)?

5. For the person described in question #4 above, what tax rate τ_3 on good 3 will minimize the excess burden of the tax system, if good 1 is not taxed, and good 2 is taxed at a rate of 50 percent?