Econ 5300 : Answers to Mid–term : F1997

1. When lump–lump transfers and taxes are possible ( case *i* ), then the utility possibility frontier slopes down : increasing one person's utility by giving her a bigger transfer requires reducing the other person's utility by increasing the lump–sum tax on him. In general, the utility possibility frontier will cross the 45–degree line, along which the two people get the same level of utility. One point on the utility possibility frontier, marked $N$ in the picture, is the result of zero taxes and transfers, the outcome of perfect competition, with each person's income coming from the sale of his or her own labour services.

The point $N$ is also on the second–best utility possibility frontier in case *ii*, since 0 taxes and transfers is certainly a feasible flat tax. Except for the point $N$, this second–best utility possibility frontier lies strictly inside the first–best ( case *i* ), since if the marginal income tax rate is positive then the allocation is not Pareto optimal. The utility possibility frontier for case *ii* must lie everywhere above the 45–degree line ( if the utility of the higher–ability person is on the vertical axis ), since the highest utility a high–ability person can achieve, given some flat tax schedule, must be at least as high as the highest utility that can be achieved by a person of lower ability ( but identical in every other respect ).

The utility possibility frontier for case *iii* also goes through $N$, since zero taxes is again a feasible income tax schedule. The *upf* in this case must again be everywhere else strictly inside the *upf* for case *i* : the optimal income tax must involve at least one person facing a non–zero marginal tax rate ( except if there are no taxes at all ), which means the outcome is again not Pareto optimal. This case *iii upf* must also lie above the 45–degree line : again, if both people must face the same income tax schedule, then the high–ability person must do at least as well. ( That is, the only way of getting below the 45–degree line is to somehow give different tax schedules to the two people. )

A flat tax is a feasible income tax for case *iii*. Therefore any utility combination possible in case *ii* must also be possible in case *iii* : the case *iii upf* must lie on or above the case *ii* u.p.f. But the optimal non–linear income tax really is non–linear ( if it is not zero ), since the optimal income tax should have a marginal rate of 0 at the top. Therefore, except at $N$, the case *iii upf* is strictly between the other 2 *upf*s.

2. In this case, differentiation of the expenditure function yields :

$$E_1 = X_1 = 2P_1^{-0.5}P_2^{0.5}u \quad E_2 = X_2 = 2P_1^{0.5}P_2^{-0.5}u$$

$$E_{11} = -(0.5)\frac{X_1}{P_1} \quad E_{12} = E_{21} = 0.5\frac{X_1}{P_2} \quad E_{22} = -0.5\frac{X_2}{P_2}$$

A commodity tax system will be optimal if and only if

$$\frac{t_1E_{11} + t_2E_{12}}{X_1} = \frac{t_1E_{12} + t_2E_{22}}{X_2}$$

When $t_1 = 0$ and $t_2 = 1$, this equation cannot be satisfied : the left side is positive and the right side is negative. In other words, given the two goods are net substitutes, taxing only good 2 increases compensated demand for good 1 and decreases compensated demand for good 2.

Here actually, the optimal commodity taxes on goods 1 and 2 are equi–proportional. ( The direct utility function is actually separable in these two goods, and the aggregator for them in that utility function is homothetic. )

A wrong tracks to take : The simple Ramsey ( inverse elasticity ) rule won't work here : that rule applies only when $E_{12} = 0$ which is certainly not the case.

3. The golden rule of economic growth is that the marginal product of capital should equal the growth rate of the labour force. Here the marginal product of capital can be written $f'(k)$ if $f(k)$ is the output per worker when $k$ is the capital stock per worker. This golden rule can be derived from maximization of the utility of a representative generation $U(C_1, C_2)$ subject to the steady–state feasibility constraint $C_1 + C_2/(1 + n) + nk \leq f(k)$.

Note that the golden rule is not necessary for the existence of the steady state. There are many other steady states possible, in which the capital per worker $k$ is constant [ given that the labour force growth rate $n$ is constant ]. Also, the golden rule is not derived by looking at the equilibrium resulting when workers of one generation hold the next generation's capital stock as their method of saving. The steady state in such a competitive equilibrium path will satisfy the golden rule only by coincidence.

In a competitive economy, if in the steady state $f'(k) < n$, then the steady–state capital stock per worker is too high. One way of reducing $k$ would be the introduction of an unfunded public pension plan. In such a plan, workers would pay taxes when young, and receive a pension when old. This plan would reduce workers' saving ( since it substitutes for private saving ). If steady–state is stable, then this reduced incentive to save will result in a lower steady–state capital stock $k$, closer to the golden rule path.

On the other hand, if $f'(k) \geq n$ in the competitive equilibrium steady state, then an unfunded public pension plan will move the economy further away from the golden rule path.