Econ 5300 : Answers to Mid–term : November 1992

1. Suppose that there are two types of individual, differing only in ability. If every person gets the same transfer income T, and faces the same proportional sales tax at the rate t, then a person of type w would have a budget constraint

$$w(1-L)+T=(1+t)x$$

where a person's type is her wage, L measures leisure (as a proportion of available time), and x is consumption. People with a higher wage have bigger choice sets ; any combination (L,x) feasible for a low–wage type will be feasible for a high–wage type (if $t<1$). Thus the high–wage type must attain higher utility if the two types' utility functions are the same, for any such tax scheme. That says that the utility possibility frontier resulting from all possible proportional taxes of this type must stay above the 45–degree line, if utility of the high–wage type is graphed along the vertical axis.

No taxes at all is possible, given that there is no exogenous government revenue requirement. No taxes at all is also a Pareto optimum to the first–best problem, in which individual lump–sum taxes can be levied. Thus the first–best and second–best utility frontiers are tangent at the "competitive" allocation resulting from no taxes at all. The second–best frontier must lie everywhere inside the first–best frontier, since any allocation possible with sales taxes is possible with lump–sum taxes.

The decision maker choosing a proportional sales tax rate will choose the no–tax allocation C if her iso–welfare curve, indicating combinations of the two types' utility that she regards as equally good, is tangent to the utility possibility frontier at C. With Benthamite preferences, the iso–welfare curves are straight lines with slopes of -1. But the first– and second–best utility frontiers are tangent to each other at C. Thus C solves the second–best problem only if it also solves the first–best problem. The example in Atkinson and Stiglitz showed that C could not solve the first–best problem when people's preferences were Cobb–Douglas and the decision maker was Benthamite, since then the slope of the first–best utility frontier (and thus the second–best utility frontier) at C was less than 1.

2. If a person maximizes a utility function

$$x-e^{aL}$$

subject to a budget constraint

$$w(1-t)L+T=x$$

where t is the income tax rate, w the wage, and T/t the exemption, then she maximizes

$$w(1-t)L + T/t - e^{aL}$$

with respect to L, yielding a first–order condition

$$w(1-t) = ae^{aL}$$

or

$$aL = \log \frac{w(1-t)}{a}$$

or

$$L = \frac{1}{a} \log \frac{w(1-t)}{a}$$

(It can be checked that the second–order condition is satisfied here.) Here labour supply is independent of exogenous income, since the utility function is quasi–linear. Therefore there is no income effect of the net wage on labour supply. Increases in the tax rate t decrease the person's labour supply ; increases in the exemption level have no effect on labour supply, provided that the person is earning more than the exempt level.

3. This question asks (obviously, I hope) about the effects of taxation in the overlapping–generations model. In that model, a tax on capital income coupled with a transfer to old people so as to balance the government's budget led unambiguously to a decrease in the steady–state capital stock.

If the utility of future generations, in the new steady state, is the criterion, then the policy is a good idea if and only if the equilibrium capital–labour ratio in the absence of the tax policy is too high, relative to the golden rule level. The golden rule, in its simplest form, prescribes that the return to capital equal the growth rate of the labour force. If that return exceeds the growth rate, then the tax policy is a bad idea, if steady–state utility is the criterion for policy evaluation.