Economics 5300.03A : Public Economics : Taxation
(Sketchy) Answers to Final Exam
January 1993

1. One of the main issues here is how the tax credit affects the effective marginal rate of taxation. The tax credit declines by 10% of any increase in the parent's income, if the income is between $10,000 and $20,000 per year. Thus a parent with several children would face a very high marginal tax rate between these income levels. The budget sets would not be convex, since this extra tax disappears at income levels above $20,000 when the tax credit has been reduced to zero.

The effect on labour supply would be a fairly substantial reduction, since the tax credit increases the parent's exogenous income, and reduces the net wage at the margin. Discussion of the simultaneous nature of the two parents' labour supply decisions would also be useful in answering this question.

2. What I wanted here was some reference to Summers's point that the two–period model will substantially understate the elasticity of the savings response to the taxation of interest income. A permanent increase in the tax rate on capital income will (ceteris paribus) decrease the net interest rate used to discount future earnings. For a 25–year–old, this will mean a very large increase in the present value of the wages she expects to earn late in her working life. This increase will lead to a large increase in current consumption, and thus a big (negative) impact on current saving.

3. This is a utility function with constant *absolute* risk aversion. Here

$$U'(W) = ae^{-aW} \qquad ; \qquad U''(W) = -a^2 e^{-aW}$$

so that the index of absolute risk aversion is a constant a.

Since a person with constant absolute risk aversion puts a constant *absolute* amount of her wealth in the risky asset as her wealth varies, a diagram such as figure 4.2.d of Atkinson and Stiglitz (page 103) shows she increases her demand for the risky asset in response to the tax.

Algebraically, she chooses her investment A inthe risky asset to maximize the expectation of $U(W) \equiv U(W_0 + r(1-t)A + r_0(1-t)(W - A))$. Her first–order condition is

$$E[e^{-aW}(r - r_0)] = 0$$

Of course, since W is itself random, this condition is certainly **not** the same as $r = r_0$. Answers which alleged that they were the same were penalized heavily. Differentiating the above expression with respect to t and A,

$$E[e^{-aW}(r - r_0)(rA + r_0(W_0 - A))]dt - (1-t)E[e^{-aW}(r - r_0)^2]dA = 0$$

which implies that

$$\frac{\partial A}{\partial t} = \frac{A}{1-t} + r_0 W_0 \frac{E[e^{-aW}(r-r_0)]}{(1-t)E[e^{-aW}(r-r_0)^2]}$$

But the second term disappears, since its numerator equals 0 from the first–order condition, so that

$$\frac{\partial A}{\partial t} = \frac{A}{1-t} > 0$$

The tax must increase demand for the risky asset.

4. This was a question about the Harberger model. Labour would bear 100 per cent of the tax if the total fall in labour's real income equalled the revenue raised by the tax. This would occur, for example, in the Harberger model when the elasticity of substitution in each industry equalled the elasticity of substitution in consumption.

5. This utility function is *not* Cobb–Douglas. It is quasi–linear ; the income elasticity of demand for all goods (except leisure) is zero. The person chooses (x, H) to maximize $U(x, H)$ subject to $q.x \leq wH$, where q is the vector of commodity prices (gross of tax). Thus she can be viewed as maximizing

$$\sum_{i=1}^{n} a_i \ln x_i - \sum_{i=1}^{n} q_i x_i / w$$

with first–order conditions

$$\frac{a_i}{x_i} = \frac{q_i}{w}$$

so that demand for each good can be written

$$x_i = \frac{a_i w}{q_i}$$

Notice the zero income elasticity of demand. Here the compensated demand for each of the goods equals the uncompensated demand. Notice also that demand for each commodity is independent of the price of any other commodity. An exact expression for the excess burden of the tax system is the sum of the areas under each demand curve between the heights of p_i and q_i, minus the tax revenue.

The person's utility if she faces prices q and a wage of w, and maximizes is

$$\sum_{i=1}^{n} \ln \frac{a_i w}{q_i} - \sum_{i=1}^{n} a_i$$

the income necessary to get her to the pre–tax utility level of

$$\sum_{i=1}^{n} \ln \frac{a_i w}{p_i} - \sum_{i=1}^{n} a_i$$

is thus

$$\sum_{i=1}^{n} a_i[\ln q_i - \ln p_i]$$

and the deadweight loss is this amount minus the tax revenue of

$$T = \sum_{i=1}^{n} \frac{a_i t_i}{q_i} w$$

This result can also be obtained directly from adding up the triangular areas under the demand curves.

6. straight out of the Stiglitz articles on optimal income taxation in the *J. Pub. Eco.* or the handbook of public economics

7. Suppose that a project costs C, and produces a stream of revenues at time u of $e^{-\delta u}x$. In the absence of tax considerations, the firm should invest in the project if and only if

$$x > (r + \delta)C$$

where r is the rate of return paid on the firms debt. The cost of capital R is defined as the rate of return R such that, when tax considerations are taken into account, the firm should invest if and only if

$$x > (R + \delta)C$$

Since borrowing costs are all tax–deductible, then the firm should discount at the rate $r(1-t)$ its future benefits. The present value of the stream of revenues, net of tax, is

$$\int_0^\infty (1-t)e^{-\delta u}e^{-r(1-t)u}x du = (1-t)\frac{x}{(1-t)r + \delta}$$

The present value of the tax deductions it gets for depreciation at the rate α is

$$\int_0^\infty te^{-\alpha u}e^{-r(1-t)u}\alpha C du = t\frac{\alpha C}{r(1-t) + \alpha}$$

So the firm should invest if and only if

$$(1-t)\frac{x}{r(1-t)+\delta} + t\frac{\alpha C}{r(1-t)+\alpha} > C$$

or (after a little re–arrangement)

$$x\frac{r(1-t)+\alpha}{r(1-t)+\delta} > C(r+\alpha)$$

which is exactly the original (no tax) condition if $\delta = \alpha$. This can be re–written

$$x > C(r+\alpha)\frac{r(1-t)+\delta}{r(1-t)+\alpha}$$

which is equivalent to

$$x > [\delta + (r+\alpha)\frac{r(1-t)}{r(1-t)+\alpha} + \frac{tr\delta}{r(1-t)+\alpha}]C$$

My definition of R, the cost of capital, is whatever makes the above condition into $x > (\delta + R)C$. That says

$$R = \frac{(1-t)(r+\alpha)+t\delta}{r(1-t)+\alpha}r$$

8. This is part of the "new" view of the property tax, propounded by Aaron and Mieszkowski (among others). To analyze the incidence of a *national* property tax, the insights of the Hraberger model are used. Housing (and commercial real estate) can be viewed as one of two sectors in the economy, all other commodities being the other sector. The housing sector produces a commodity, housing services, using land, labour and capital as inputs. A national property tax is then viewed as an excise tax on housing services.

Here capital is assumed perfectly mobile between sectors, and inelastic in aggregate supply. Then capital owners in general (not in one sector or the other) will bear (much of) the property tax, if there were only two factors of production, and if the housing sector were relatively capital intensive.

Of course the algebra gets more complicated when there are three factors of production. And the very nature of the Harberger model directs attention from any incidence issues on the consumption side.

Moreover, the property tax is not a national tax, it is a local tax. The above analysis may apply to the *average* property tax rate. But there are then excise tax effects associated with each jurisdiction's deviation (positive or negative) from this national average. These excise tax effects may be born by consumers of housing, as in the "old" view.