GS/ECON 5300 Answers to Midterm Exam October 2003

1. To determine the person's saving behavior, note that she maximizes her utility subject to the budget constraint

$$(1+r(1-t))(Y-C_1) \geq C_2$$

where r is the rate of return on saving, t is the tax rate on saving, and Y is her income (all earned in the first period).

So the Lagrangean is

$$A\sqrt{C_1} + C_2 + \lambda(1+r(1-t))(Y-C_1) - C_2)$$

yielding first–order conditions

$$\frac{A}{2\sqrt{C_1}} = \lambda(1+r(1-t)) \quad (1)$$

$$1 = \lambda \quad (2)$$

Substituting for λ from equation (2) in equation (1) gives

$$2\sqrt{C_1} = \frac{A}{r(1-t)}$$

or

$$C_1 = \frac{A^2}{4(1+r(1-t))^2}$$

Since savings is

$$S = Y - C_1$$

therefore

$$S = Y - \frac{A^2}{4(1+r(1-t))^2}$$

Since the utility function is quasi–linear, the demand for current consumption C_1 does not depend on the person's income Y. The income elasticity of current consumption C_1 is zero. So there is no income effect, and the substitution effect must dominate. Therefore, a tax on the return to saving must reduce the person's saving, from

$$Y - \frac{A^2}{4(1+r)^2}$$

to

$$Y - \frac{A^2}{4(1+0.75r)^2}$$

2. If there is some possibility that the risky asset might earn a negative return, then the fact that there is no loss offset matters. For example, suppose that there were two possible states of the world, and that the possible returns on the (only) risky asset were r_b and r_g, with $r_b < 0 \leq r_s < r_g$,

where r_s is the return on the safe asset. With no tax, if the person invests A (out of a total wealth of W) in the risky asset, then she will have wealth of

$$W_g = (1 + r_g)A + (1 + r_s)(1 - A)$$

in the good state, and

$$W_b = (1 + r_b)A + (1 + r_s)(1 - A)$$

So that $\partial W_g/\partial A = r_g - r_s$, $\partial W_b/\partial A = r_b - r_s$, and the slope of her budget line (with W_g on the horizontal, W_b on the vertical) is

$$\frac{dW_b}{dW_g} = -\frac{r_g - r_s}{r_s - r_b}$$

taxing the return to saving at the rate t will now change W_g and W_b to

$$W_g = (1 + r_g(1 - t))A + (1 + r_s(1 - t))(1 - A)$$

$$W_b = (1 + r_b)A + (1 + (1 - t)r_s)(1 - A)$$

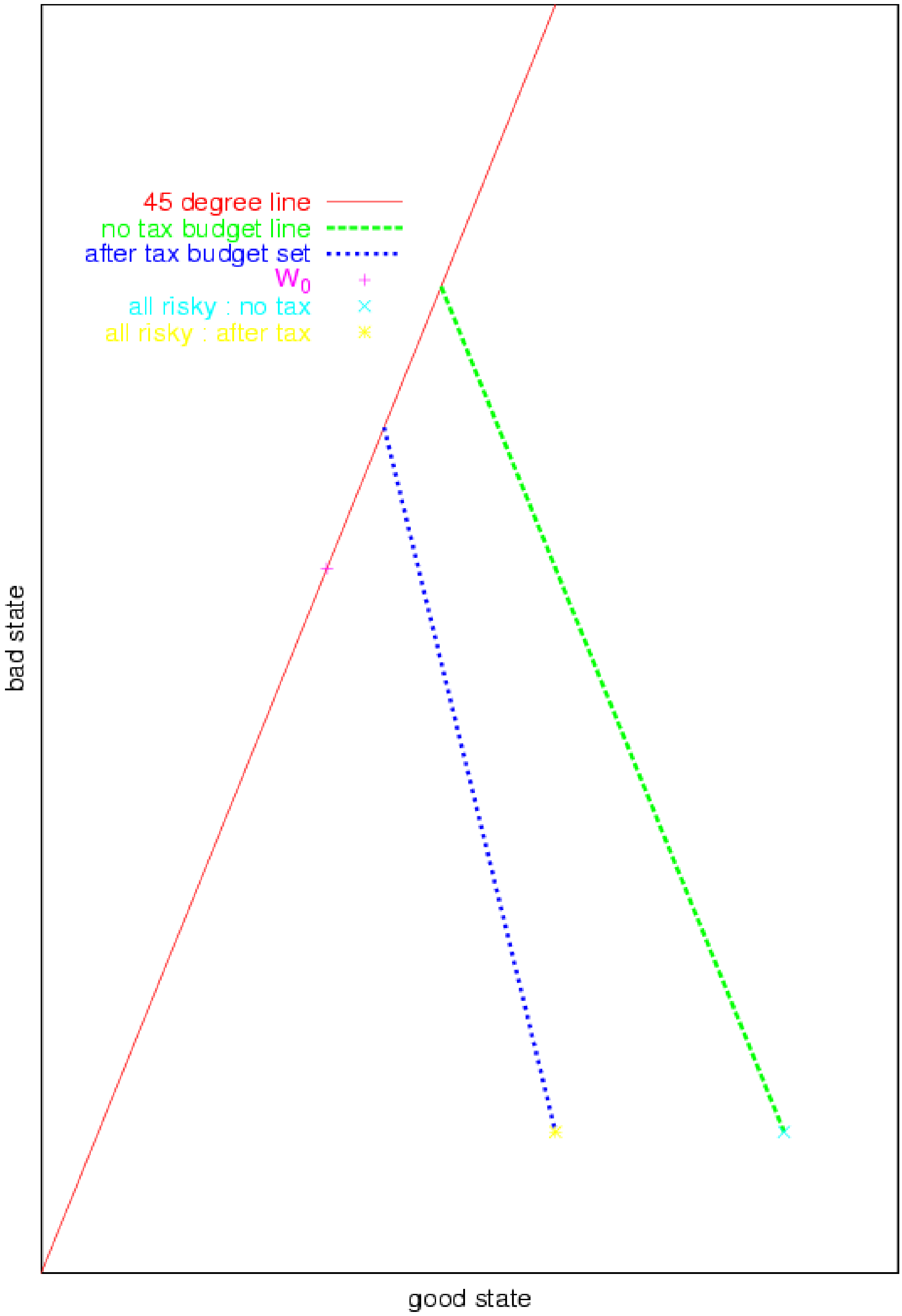
if there is no loss offset. Now the slope of the budget line is

$$\frac{dW_b}{dW_g} = -\frac{r_g - r_s}{r_s - r_b/(1 - t)}$$

The absence of loss offset means that the budget line has become steeper.

So investment in the risky asset is being discouraged, compared to a tax with no loss offset. The “income effect” of a parallel shift of the budget line, which occurs when there is full loss offset, is now modified by a “substitution effect”, since the budget line has become steeper. The person may decrease her investment A in the risky asset, even if she had, for example, a constant coefficient of relative risk aversion.

(The picture illustrates the case in which $r_g = 0.4$, $r_s = 0.1$ and $r_b = -0.2$. If the elasticity of substitution were high enough,the tax would reduce the person’s investment inthe risky asset if there were no loss offset.)


45 degree line
no tax budget line
after tax budget set
W_0
all risky : no tax
all risky : after tax
bad state
good state

3. In a simple 2–period overlapping–generations model, people work only in the first period, but consume in both periods of their lifetime. So an equal–yield shift from labour income taxation to consumption taxation would mean taxing people less when they were young workers, and more when they were old retired people.

In other words, it is exactly like a transfer to workers, financed by a tax on retired people. In the steady state of an overlapping–generations model, such a change must increase capital accumulation, since it encourages more saving. The change in the steady–state capital stock k per worker could be written

$$dk = \frac{1 - C_W^1}{A} dT^1 - \frac{C_W^1}{A(1+r)} dT^2$$

where C_W^1 s the marginal propensity to consume in the first period, r is the net return to saving, dT^i is the change in the transfer to people in period i of their life, and A is an expression which must be positive. So if the income derivative of first–period consumption lies between 0 and 1, then the tax change, which increases T^1, the net transfer paid to young workers, and decreases T^2, the transfer paid to older retired people, must increase the steady–state level of capital per worker.