GS/ECON 5300 Public Economics I October 2001

Answers to Mid–term

1. The best answer is to take the definition of the excess burden

$$L(p,t,u) \equiv E(p+t,u) - E(p,u) - t \cdot X^c(p+t,u)$$

where p is the vector of pre–tax prices, t is the vector of unit taxes, u is the original (pre–tax) level of utility, and $X^c(\cdot,u)$ is the vector of the person's compensated demands. $E(p,u)$ is the person's expenditure function, defined as the cost of the lowest–cost consumption bundle which gives her a utility level of u, when the price vector is p.

From this definition of the expenditure function

$$E(p,u) = p \cdot X^c(p,u)$$

and

$$E(p+t,u) = (p+t) \cdot X^c(p+t,u)$$

since the compensated demand function is the vector of quantities of goods which minimize the cost of the given level of utility.

Therefore

$$\begin{aligned} L(p,t,u) =&(p+t) \cdot X^c(p+t,u) - p \cdot X^c(p,u) - t \cdot X^c(p+t,u) \\ =&p \cdot X^c(p+t,u) - p \cdot X^c(p,u) \end{aligned}$$

$X^c(p,u)$ is the bundle which minimizes the cost of attaining a utility level u, when prices are p. Therefore any other consumption bundle which achieves the same level of utility must cost more than $p \cdot X^c(p,u)$. In particular, the bundle $X^c(p+t,u)$ achieves the same level of utility u. So — at prices p — $X^c(p+t,u)$ costs more than the bundle $X^c(p,u)$, so that

$$p \cdot X^c(p+t,u) > p \cdot X^c(p,u)$$

The only time that the above inequality would be an equality is if the two consumption bundles $X^c(p+t,u)$ and $X^c(p,u)$ were exactly the same. That would be the case only if the person did no substitution of one good for another when relative prices changed, or if none of the relative prices were changed by the taxes.

An alternative, not as general, is to assume that the second derivatives of the person's expenditure function were all constant. In this special case,

$$L(p,t,u) = -t'Et$$

where E is the matrix of second derivatives of the expenditure function. Since this matrix is negative semi–definite, $t'Et \leq 0$ for any tax vector t, and will be 0 only if the vector of taxes is proportional to the vector of prices.

The diagram in the Diamond–McFadden paper also illustrates the excess burden when leisure is untaxed, by lumping "all other goods" into a single composite good.

2. Taking the derivatives of the expenditure function,

$$E_1(q_1, q_2, w, u) = -\frac{w^2}{q_1^2}$$

$$E_2(q_1, q_2, w, u) = \frac{w}{q_2}$$

So that

$$E_{11}(q_1, q_2, w, u) = -2\frac{w^2}{q_1^3}$$

$$E_{22}(q_1, q_2, w, u) = -\frac{w}{q_2^2}$$

$$E_{12}(q_1, q_2, w, u) = E_{21}(q_1, q_2, w, u) = 0$$

Since the cross–partials of the compensated demand functions are zero, the "inverse elasticity" version of the Ramsey rule can be applied : the optimal tax rates should be inversely proportional to the compensated own–price elasticities of demand.

It is straightforward to calculate those elasticities. The compensated own–price elasticity is the derivative of compensated demand for a good with respect to the good's own price, times the price, divided by the quantity. Here E_i is the quantity, and E_{ii} is the derivative of quantity with respect to the price. So

$$\eta_1 = -E_{11}\frac{q_1}{E_1} = 2\frac{w^2}{q_1^3}q_1\frac{q_1^2}{w^2} = 2$$

$$\eta_2 = -E_{22}\frac{q_2}{E_2} = \frac{w}{q_2^2}q_2\frac{q_2}{w} = 1$$

Since the compensated elasticities are not equal, then the two goods should not be taxed at the same rate.

The more general form of the Ramsey rule can be used here. The vector of unit taxes (t_1, t_2) will be optimal if and only if

$$\frac{t_1E_{11} + t_2E_{12}}{E_1} = \frac{t_1E_{12} + t_2E_{22}}{E_2}$$

If the two goods were taxed at the same rate, then we would have $t_1 = \tau q_1$ and $t_2 = \tau q_2$ where τ is the common tax rate. Therefore, here the uniform tax system would be optimal if and only if

$$\tau\frac{q_1E_{11}}{E_1} = \tau\frac{q_2E_{22}}{E_2}$$

and the computation above shows that this equality does not hold : the left side is twice as large as the right side.

(Of course, if w were increased by the same proportion as q_1 and q_2, then the tax system would have no excess burden at all. But that would involve a wage *subsidy*. And the Ramsey formula is not needed to demonstrate that it's efficient to raise all prices by the same fraction.)

3. If a tax is imposed on saving —or if a tax on saving is removed — there will be both an income effect and a substitution effect. If first–period consumption is a normal good, than these effects work in offsetting directions, so that the overall effect of a tax change on the person's saving (if the amount of the person's saving was positive) would be ambiguous.

However the question did not ask about simply removing the taxation of the return to saving. It asked about removing that tax, and at the same time introducing a head tax which left the person on the same indifference curve as before. In other words, the effect of this policy change is a pure substitution effect — a move along the indifference curve.

That means that the policy change must *increase* the person's saving in the standard two–period model (with exogenous income in each period), since the substitution effect of an increase in the return to saving is to unambiguously decrease first–period consumption and increase saving.

(If the person's income in each period also were endogenous, then even this pure substitution effect could not be signed.)