GS/ECON 5010 APPLIED MICROECONOMICS

Answers to Midterm Exam Winter 2005

*Q*1. If a person's preferences are strictly convex, show why there is a unique consumption bundle which is preferred to any other consumption bundle in her budget set.

*A*1. The easiest, and most direct method, is to go straight to the formal definition of strict convexity of preferences : if a person's preferences are strictly convex, then if she is indifferent between two distinct consumption bundles $\mathbf{x}^1$ and $\mathbf{x}^2$, she will prefer strictly any other bundle $\mathbf{x}^3$ which is a convex combination of $\mathbf{x}^1$ and $\mathbf{x}^2$, that is any

$$\mathbf{x}^3 = t\mathbf{x}^1 + (1-t)\mathbf{x}^2$$

with $0 < t < 1$.

So suppose that there were two distinct consumption bundles $\mathbf{x}^1$ and $\mathbf{x}^2$ which were tied for the best, among all consumption bundles in the person's budget set. Any $\mathbf{x}^3$ which is a convex combination of these two bundles will be preferred strictly to either of them. And the bundle $\mathbf{x}^3$ will also be in the person's budget set, since

$$\mathbf{p} \cdot \mathbf{x}^3 = t\mathbf{p} \cdot \mathbf{x}^1 + (1-t)\mathbf{p} \cdot \mathbf{x}^2 \leq ty + (1-t)y = y$$

This means that two (or more) bundles cannot be tied for best : if they were, then a bundle such as $\mathbf{x}^3$ would be strictly preferred to both (and in the budget set), so that the two tied bundles would not be the best.

*Q*2. If a person's utility–of–wealth function is

$$u(W) = \sqrt{W}$$

calculate the risk premium associated with an investment project which will double the person's wealth with probability 0.5, and which will leave the person's wealth unchanged with probability 0.5.

*A*2. The person's expected utility from the project is

$$EU = (0.5)\sqrt{W} + (0.5)\sqrt{2W}$$

so that the certainty equivalent CE to the project is the amount of money CE such that

$$u(W + CE) = EU$$

That means that CE is the solution to the equation

$$\sqrt{W+CE} = (0.5)\sqrt{W} + (0.5)\sqrt{2W} \qquad (2-1)$$

Multiplying both sides by 2,

$$2\sqrt{W+CE} = \sqrt{W} + \sqrt{2W} \qquad (2-2)$$

Squaring both sides of equation $(2-2)$,

$$4(W+CE) = W + 2\sqrt{W}\sqrt{2W} + 2W \qquad (2-3)$$

or

$$4(W+CE) = [3+2\sqrt{2}]W \qquad (2-4)$$

implying that

$$CE = \frac{1}{4}[2\sqrt{2}-1]W \qquad (2-5)$$

The expected value EV of the project is $W/2$, since it has a return of W with probability (0.5) and a return of 0 with probability 0.5.

The risk premium is the difference between the expected value of the project, and the certainty equivalent, $EV-CE$. So the risk premium is

$$\frac{W}{2} - \frac{1}{4}[2\sqrt{2}-1]W = \frac{1}{4}[3-2\sqrt{2}]W \approx (0.045)W \qquad (2-6)$$

$Q3$. Calculate the cost function, if the production function is

$$y = f(x_1, x_2) = 100 - \frac{1}{x_1} - \frac{1}{x_2}$$

$A3$. Cost minimization implies that the firm should use a mix of the two inputs such that

$$\frac{f_1}{f_2} = \frac{w_1}{w_2} \qquad (3-1)$$

where f_i is the marginal product of input i, and w_i is the price of input i.

Here

$$f_i = \frac{1}{x_i^2}$$

so that the cost minimization condition $(3-1)$ becomes

$$(\frac{x_2}{x_1})^2 = \frac{w_1}{w_2} \qquad (3-2)$$

or

$$x_2 = \sqrt{\frac{w_1}{w_2}}x_1 \qquad (3-3)$$

Substituting from $(3-3)$ into the production function,

$$y = 100 - \frac{1}{x_1} - \sqrt{\frac{w_2}{w_1}}\frac{1}{x_1} \qquad (3-4)$$

or

$$\frac{1}{x_1}\frac{\sqrt{w_1}+\sqrt{w_2}}{\sqrt{w_1}} = 100 - y \qquad (3-5)$$

so that the conditional input demand for input 1 is

$$x_1 = \frac{\sqrt{w_1}+\sqrt{w_2}}{\sqrt{w_1}}\frac{1}{100-y} \qquad (3-6)$$

Analogously (or from substitution from equation $(3-3)$), the conditional input demand for input 2 is

$$x_2 = \frac{\sqrt{w_1}+\sqrt{w_2}}{\sqrt{w_2}}\frac{1}{100-y} \qquad (3-7)$$

The cost function is then just the cost $w_1 x_1(\mathbf{w}, y) + w_2 x_2(\mathbf{w}, y)$ of this cost–minimizing bundle. From equations $(3-6)$ and $(3-7)$,

$$C(\mathbf{w}, y) = w_1 \frac{\sqrt{w_1}+\sqrt{w_2}}{\sqrt{w_1}}\frac{1}{100-y} + w_2 \frac{\sqrt{w_1}+\sqrt{w_2}}{\sqrt{w_2}}\frac{1}{100-y} = [\sqrt{w_1}+\sqrt{w_2}]^2 \frac{1}{100-y} \qquad (3-8)$$

(You can check that Shepherd's Lemma holds : $\frac{\partial C}{\partial w_i} = x_i$.

*Q*4. Would the own–price elasticity of demand for the product of a single–price monopoly ever be greater than 1 in absolute value? Explain briefly.

*A*4. The answer is "yes". The own–price elasticity of demand **must** exceed 1 in absolute value, if the monopoly is maximizing profit. If the elasticity were less than 1, then the monopoly could increase profits by reducing output. This reduction would lower costs. It would also raise sales revenue, since a reduction in quantity sold (and the associated increase in the price of the good) will raise revenue when demand is inelastic. So an elasticity less than 1 is inconsistent with profit maximization.

Formally, the first order condition for profit maximization by a single–price monopoly is

$$p(1 - \frac{1}{\epsilon}) = MC$$

where p is the price, MC the marginal cost, and ϵ the absolute value of the own–price elasticity of demand. Given that p and MC are both positive, this condition can hold only if $\epsilon > 1$.