

due : Wednesday March 9    before class

Do all 5 questions. Each counts 20%.

1. What are the market price, and aggregate quantity sold, in long-run equilibrium in a perfectly competitive market for which the demand function has the equation

$$Q = 2000 - 5p$$

(where  $Q$  is aggregate quantity demanded, and  $p$  the price), if there is free entry by identical firms to the industry, each of which has the long-run total cost function

$$TC = 600q - 80q^2 + 4q^3$$

where  $q$  is the quantity produced by the firm?

2. Suppose that a market contains 1 million identical consumers, each of whom has preferences which can be represented by the utility function

$$U(X, q_1, q_2) = X + (q_1^\alpha + q_2^\alpha)^\beta$$

where  $X$  is consumption of a numéraire good, and  $q_1$  and  $q_2$  are consumption of goods produced by firms #1 and #2 respectively, and where  $\alpha < 1$  and  $\beta < 1$ .

Firms 1 and 2 each have the identical total cost function,  $C(q, \mathbf{w}) = cq$ , where  $c$  is a constant.

*i* Which levels of output for the two firms would maximize their combined profits?

*ii* What levels of output would firms #1 and #2 produce if they behaved as Cournot duopolists?

(You may assume that  $q_1 = q_2$  in the solutions to each of the problems.)

3. Solve for the quantity produced by each firm, the price, and each firm's profits, if there were  $J$  firms acting as Cournot oligopolists, each producing a homogeneous good, for which the market demand is linear

$$p = a - bQ$$

where  $Q \equiv q_1 + q_2 + \dots + q_J$  was industry output, if each firm had the (same) total cost function

$$C(q) = cq^2$$

for some positive constant  $c$ ?

4. What would be the equilibrium price, and the equilibrium profits of each firm, in a market with two Bertrand oligopolists, producing goods which are imperfect substitutes for each other, with quantity demanded of the products of the two firms being

$$q_1 = \frac{p_1^{r-1}}{p_1^r + p_2^r}$$

$$q_2 = \frac{p_2^{r-1}}{p_1^r + p_2^r}$$

where  $r < 0$ , if each firm's cost of producing  $q$  units is  $cq$  (where  $c$  is a positive constant). (You may restrict attention to symmetric equilibria, in which  $p_1 = p_2$ .)

5. Another model of duopoly is that of **von Stackelberg**, in which firms choose output levels **sequentially**. That is, firm 1 chooses its output. Firm 2 observes what output level firm 1 has chosen, and then chooses its own output level. What output levels would the 2 firms choose, if they behaved in this manner, if they both produced an identical product for which the market inverse demand function had the equation

$$p = 21 - (q_1 + q_2)$$

if each firm had a total cost function

$$TC = \begin{cases} 4 + q_i & \text{if } q_i > 0 \\ 0 & \text{if } q_i = 0 \end{cases}$$

where  $q_i$  is the output level of firm  $i$ ?