First-Price and Second-Price Auctions with Resale

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Abstract

In the independent-private-value model, I add a resale stage to standard sealed-bid auctions with two, possibly heterogeneous, bidders. The bids are either kept secret or made public after the auction. Either the auction winner or the auction loser proposes the resale price. Because the bidders' endogeneous net values for winning the auction are identical for identical bids, the methods previously developed for the pure-common-value model can be applied. I characterize an infinity of equilibria of the second-price auction and a unique equilibrium of the first-price auction. For every equilibrium of an auction with secret bids, I construct an outcome-equivalent and, in the case of the second-price auction, "posterior implementable" equilibrium of the same auction with public bids. I study how the auctioneer's expected revenues are affected by a change of the auction or of the bargaining procedure at resale. I extend my results to a class of auctions that includes the k-price auctions.

1. Introduction

Most theoretical auction models forbid resale between bidders. However, resale is at least possible after many real-life auctions. Documented examples

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include auctions of bonds, bills, foreign exchange, timber rights, pollution allowances, radio-wave spectrum licences, real-estate, and gold.

To standard sealed-bid auctions in the independent-private-value model with possibly (ex ante) heterogeneous bidders, whose consumption values are distributed differently, I add a stage where resale under incomplete information may take place between bidders. Only information pertaining to the auction outcomes–the bids and the auction winner–may become available before the resale stage. In particular, no exogenous signal correlated with the bidders' private information is revealed. I consider two regimes regarding the disclosure of the bids: full disclosure, when the auctioneer reveals all bids before resale or, equivalently, when bids are submitted publicly; and no disclosure, when the auctioneer defers the auction payments and the revelation of the bids until after resale. My equilibria in the full-disclosure regime are actually also equilibria in the no-disclosure regime, as well as in intermediate regimes where, for example, only the highest bid is revealed, as in the Dutch auction, or only the losing bid is revealed, as in the English auction. The bidders are risk-neutral and do not discount their future payoffs[2].

I devote most of the paper to the second-price auction, or SPA, and the first-price auction, or FPA, with two bidders where bargaining at resale takes the simple form of a take-it-or-leave-it offer from either the auction winner or the auction loser. I also show how some of my results extend to two-bidder (k_1, k_2)-price auctions, which are hybrids between the FPA and the SPA and generalize k-price auctions.

Under the full-disclosure regime, bidders engage in signalling through their bids at auction. No perfect Bayesian equilibrium, or PBE, where the bidders follow different strictly increasing (and differentiable) bidding functions exists. If there existed such an asymmetric separating equilibrium, there would exist a domain of consumption values where a bidder would be

[2] In Footnote 10 I show how the results can be extended to the case of arbitrary common discounting with no consumption between auction and resale.

"less aggressive," that is, would follow a strictly smaller bidding function. When this bidder's value belongs to this domain, inefficient allocations and profitable resale could only occur when he loses the auction. Assume, for example, the winner at auction is the resale-price maker. Then, this (locally) "less aggressive" bidder could only be the price taker at resale–he would be the "equilibrium resale-price taker"–and would be left with no resale surplus if he revealed his true value through his equilibrium bid. Instead, when he would have no incentive to increase his bid, he would always find more advantageous to lower it in order to misrepresent his value and improve his terms of trade at resale (see Section 3).

Under full disclosure, no fully-separating PBE then exists in the FPA with heterogeneous bidders, since only different bidding functions could possibly satisfy the bidders'–different–first-order conditions. In the SPA under full disclosure with arbitrary bidders, the only separating PBE is the only symmetric separating PBE: the truth-bidding PBE. However, contrary to the standard SPA where resale is not allowed, truth-bidding is not a weakly dominant strategy (on this point, see, for example, Gupta and Lebrun, 1999).

Under no disclosure, because bidders cannot signal through their bids, there do exist one pure separating PBE of the FPA and a multiplicity of such equilibria of the SPA. Surprisingly, for every PBE under no disclosure, there exists a PBE under full disclosure that generates the same bid probability distributions, probabilities of trade conditional on the bid of the equilibrium resale-price maker, resale prices, bidders' interim expected payoffs, and auctioneer's expected revenues.

The asymmetric PBE's under full disclosure are "semi-separating" equilibria, where the equilibrium resale-price taker, by randomizing over a range of bids, does not perfectly reveal his value through his bid. All the PBE's of the SPA are "posterior implementable," that is, are such that no bidder regrets his bid after learning his opponent's bid. Any of the PBE's of the SPA is also robust to some changes in the value distributions.

The similarity between the numbers of equilibria–one in the FPA and an infinity in the SPA–in my model with resale and in the standard two-bidder pure-common-value model (see the next section for references) is not accidental. Indeed, I show that these two models are intimately linked. The link would be immediate if all private information was exogenously released before resale. In this case, resale would correct any inefficient auction outcome and both bidders' net values for winning the auction would be equal to the resale price. In fact, if the winner would be a seller at resale, his revenues from his intermediation between the auctioneer and the other bidder would be the resale price and, if he would be a buyer at resale, his savings from buying at auction rather than at resale would also be the resale price.

However, in the present model, no private information is exogenously released, and, because of the remaining incomplete information, resale does not remedy all inefficient outcomes. A private component thus enters the bidders' net values for winning and the model is not formally equivalent to the pure common-value model. A further difference with the standard pure-common-value model is the endogeneity of these net values. The resale-price maker's revised beliefs and hence his optimal offer depend on the other bidder's equilibrium bidding strategy. Under the full-disclosure regime, he conditions his beliefs on the bid from his opponent and under the no-disclosure regime, if, for example, he is the auction winner, on the information that his opponent's bid was smaller than his.

Nevertheless, the pure-common-value model is relevant as long as the bidders' net values for winning are equal when their bids are identical, because these net values determine the incentives to marginally change the bids. In the no-disclosure regime, if the bids are identical and the auction outcome is inefficient, resale does take place and the bidders have the same net value (the resale price) for winning. Consider, for example, the case where the auction winner is the resale-price maker. The consumption value of the auction loser, if he has submitted the same bid, is the highest value consistent with losing–the upper extremity of the support of the auction winner's revised beliefs. If, according to these revised beliefs, profitable resale is possible, the auction winner sets a resale price strictly below the upper extremity and hence the auction loser agrees to trade.

Thanks to the equality between the bidders' net values for identical bids, and a couple of other properties my model shares with the pure-common-value model, the pure equilibrium strategies under the no-disclosure regime are actually equilibrium strategies in a pure-common-value model if the bidders' consumption values are interpreted as private signals that determine, according to an endogeneous function, the bidders' common value. For example, when the auction winner sets the resale price, this common value is the auction winner's optimal resale price as a function of his consumption value and the upper extremity of the truncated interval of the auction loser's consumption values.

I compare, for a given bargaining procedure at resale, the auctioneer's expected revenues from different auction formats. I show that with homogeneous bidders, whose consumption values are identically distributed, the PBE of the FPA and the truth-bidding PBE of the SPA bring the same revenues, which are higher than the revenues from any other of the PBE's of the SPA. In fact, the symmetry of these two PBE's imply their optimality among all mechanisms that sell the good with certainty. With heterogeneous bidders, the PBE of the FPA is not optimal. Some PBE's of the SPA give higher revenues, and some others give lower revenues.

I also study the effect of a change of bargaining procedure at resale on the PBE's of a given auction format. I show that, contrary to the simple model with resale under complete information in Gupta and Lebrun (1999), changing the resale-price maker from the auction loser to the auction winner does not shift upward the equilibrium bid distributions in the FPA. I prove that it nevertheless increases the auctioneer's expected revenues. In the SPA, I establish that changing the bargaining at resale does not affect the range of equilibrium revenues. In fact, under assumptions similar to Zheng (2002)'s, I prove that the upper extremity of this range is equal to the revenues from the optimal mechanism that allocates the item with certainty.

I show how the results can be extended from the FPA to any k-price auction, or k-PA, where the price is the weighted average of the highest bid and the second highest bid with respective weights k and $1-k$, with $0 < k \leq 1$. They can also be extended to (k_1, k_2)-price auctions, or (k_1, k_2)-PA's, where the weights may depend on the identity of the auction winner. I prove that the PBE's of the (k_1, k_2)-PA and the (k_1', k_2')-PA are payoff-equivalent if and only if $k_1/k_2 = k_1'/k_2'$. I also show that the limit of PBE's of any (k_1, k_2)-PA's, when (k_1, k_2) tends towards $(0, 0)$ while the ratio k_1/k_2 is fixed, is one of the PBE's of the SPA.

I examine the relations with the literature in the next section. I first address, in Sections 3 to 6, the bargaining procedure at resale where the auction winner sets the price. I offer a characterization of the PBE's in the no-disclosure regime in Section 3, prove it in Section 4, and go on to establish their useful properties in Section 5. Section 6 is devoted to the full-disclosure regime and is divided into two subsections: Subsection 6.1, where I state the equivalence with the no-disclosure regime, and Subsection 6.2, where I prove it. I then obtain the results for the other bargaining procedure at resale, where the auction loser sets the price, in Section 7 and Appendix 3. I gather the results of comparative statics in Section 8 and extend the results to (k_1, k_2)-PA's in Section 9. Section 10 concludes. Details of some proofs, examples, and further extensions can be found in the appendices.

2. Relations with the Literature

Often in my proofs, I rely on forms of my model that are amenable to the methods developed for pure-common-value auctions with two bidders. These methods for the SPA were introduced and used by Milgrom (1981), Bikhchandani (1988), Bikhchandani and Riley (1991), Klemperer (1998), and Bulow, Huang, and Klemperer (1999). The FPA with pure common value is studied in Wilson (1967, 1969, 1977), Ortega-Reichert (1968), Rothkopf (1969), Reece (1978), Milgrom (1979 a, b), Milgrom and Weber (1982 a), Engelbrecht-Wiggans, Milgrom, and Weber (1983), and Parreiras (2006). The model of pure common value is a special case of the model of affiliation Milgrom and Weber (1982 b) introduced.

The auction loser's expected payoff at resale is the result of an endogeneous positive externality of the other bidder's winning the auction. Auctions with exogenous externalities are investigated in Jehiel, Moldovanu, and Stachetti (1996, 1999), Jehiel and Moldovanu (2000), and Das Varma (2002).

I obtain equilibria when either the auction winner or the auction loser has the whole bargaining power at resale. Kamien, Li, and Samet (1989) consider, in a complete information model, Bertrand duopolists who compete for a contract and bargain, under the same procedures, for the terms of the subcontract.

Although not fully separating, all my asymmetric equilibria under the full-disclosure regime display some signalling through the bids at auction. Such signalling already appeared in Ortega-Reichert (1968) and Hausch (1986), in the context of a sequential multi-unit auction, and in Whaerer (1999), Goeree (2002), and Katzman and Rhodes-Kropf (2002), in the context of a one-unit auction followed by strategic interactions, such as Cournot or Bertrand competition between the bidders or related bargaining between the auction winner and the auctioneer.

Under the assumption of complete information throughout, Milgrom (1987) and, for two-unit sequential auctions, Campos e Cunha and Santos (1995) and Gale, Hausch, and Stegeman (2000) study resale in auctions. Ausubel and Cramton (1999) consider a model of auctions followed by resale that they assume to be ex-post efficient. In Haile (2000, 2001, 2003), bidders at auction are uncertain about their own consumption values. Haile (2000,

2001) assumes the consumption values become public knowledge before resale, which takes place under complete information. Haile (2003) also considers the model where only the private uncertainty is lifted before resale, that is, where a bidder's consumption value becomes known to him and not to others, and obtains expressions for the equilibria, conditional on their existence. Furthermore, Haile (2003) compares different auction formats and bargaining procedures at resale.

In the independent-private-value model of the FPA with two bidders, Gupta and Lebrun (1999) also assume that information becomes complete before resale and consider arbitrary exogeneous resale-price functions. Through the net values for winning, their model then reduces to a pure-common-value model. I explain in Section 4 that, due to the incomplete information at resale, my present model is not equivalent to a pure-common-value model.

Cheng and Tan (2007) appeal to my characterization of the equilibrium of the FPA with resale under the no-disclosure regime and also note that its bidding strategies form an equilibrium of a pure-common-value model. Here, I go further since I actually isolate (in Lemmas 2 and 5, Sections 4 and 6) the properties that the two models share and use them to prove my results, including the characterization of the equilibria under both regimes.

Garratt and Tröger (2006) add to the bidders of a symmetric independent-private-value model one "speculator," whose has no consumption value for the item, and allow for discounting. They describe an infinite class of equilibria for the SPA and one equilibrium for the FPA. The results for the SPA are extended in Garratt, Tröger, and Zheng (2006) to the two-bidder case where all biders have strictly positive consumption values (they also describe n "extreme" equilibria in the n-bidder case). In Appendix 6, I show how their equilibria of the SPA without discounting can be obtained from a (proper) subset of the equilibria in the present paper.

Bose and Deltas (2002, 2004) show the presence of a winner's curse in auctions between several speculators and one final consumer, although they do not allow the speculators to act on the information they gather at auction. Bikhchandani and Huang (1989, 1993) consider a common-value model with affiliated signals where the bidders are speculators who, after winning, resell the item to final consumers with no private information.

Pagnozzi (2007 a) shows that resale may occur at the equilibrium of a SPA that awards a project with random cost to one of two heterogenous bidders, one with limited liability and none with private information. Pagnozzi (2007 b) studies the effects of resale and allowing speculators in a multi-unit auction when bidders have flat demands and information is complete.

The model of the present paper is closest to the model of Krishna (Chapter 4, 2005) and Hafalir and Krishna (2007 a, b), where resale follows an auction between two bidders with independent values that remain private all along. Krishna (2005) shows, in Proposition 4.6, p. 58, that, if the average consumption values differ across bidders, there exists no pure equilibrium of the FPA under the full disclosure regime that results in a post-resale efficient allocation. Hafalir and Krishna (2007 a, b) focus on the no-disclosure regime (or equivalent combinations of auction and resale formats). Hafalir and Krishna (2007 a) obtain the same unique pure equilibria of the FPA I obtain in Theorem 2 (Section 3) and Theorem A1 (Appendix 3) for the two bargaining procedures at resale under this regime[3]. Using a tool from the calculus of variation, they show that, if the bidders are heterogeneous, the equilibrium of the FPA gives higher expected revenues than the truth-bidding equilibrium of the SPA. Although, as Hafalir and Krishna (2007 a) notice, the truth-bidding equilibrium of the SPA is its only ex-post equilibrium, I show in Section 6, that there exists an infinity of posterior implementable equilibria of the SPA under the full-disclosure regime, some giving higher and others lower revenues than the equilibrium of the FPA. I also show in Section 6 how to construct an equivalent behavioral equilibrium of the FPA under

[3]In addition to the two ultimatum procedures, Hafalir and Krishna (2007 a) consider "probabilistic" procedures where the resale-price maker is chosen at random according to exogeneous probabilities.

full disclosure from its equilibrium under no disclosure[4]. For two classes of value distributions for which the equilibria of the FPA without resale can be characterized explicitly, Hafalir and Krishna (2007 b) compares the FPA's with and without resale.

In Section 8, I present assumptions–Assumption A^s, when the auction winner sets the resale price, and Assumption A^b, when the auction loser sets the resale price–under which equilibria of the SPA give revenues that approach the highest possible revenues from any selling mechanism that never retains the item. Zheng (2002), in the standard independent-private-value model with n bidders, and Calzolari and Pavan (2006), in a discrete two-bidder model, address the issue of optimal mechanism design when resale is possible. My Assumption A^s is an extension, in the two-bidder case, of Zheng (2002)'s assumptions. With identity-dependent bargaining powers, Calzolari and Pavan (2006) find that the constrained optimal mechanism with resale can be implemented as auctions[5]. In a common-value model, Mares (2005) obtains the optimal mechanism as an equilibrium of the SPA, which he selects among the multiplicity of equilibria of the SPA[6].

In Section 9, I analyze k-PA's and their discriminatory extensions, (k_1, k_2)-PA's, with resale and show that, as they approach the SPA, their equilibria approach some of the equilibria of the SPA. Güth and van Damme (1986) first introduced k-PA's. In a pure-common-value model with two bidders and affiliated signals, Parreiras (2006) selects, among the infinity of equilibria of the SPA, the equilibrium that is the limit of equilibria of k-PA's, as k

[4]The standard FPA with heterogeneous bidders and without resale is studied in, for example, Maskin and Riley (2000 a and b), for the two-bidder case, and Lebrun (1999), for the n-bidder case.

[5]Depending on the values of their parameters, these auctions are FPA's with limited sets of acceptable bids (and specific tie-breaking rules) or SPA's with personalized reserve prices.

[6]If I applied Mares (2005)'s selection criterion to my model, I would obtain the revenue-superiority of the SPA over the FPA when bidders are heterogeneous. See also the next footnote.

tends towards zero[7].

3. The No-Disclosure Regime

One item is being auctioned to one of two risk-neutral bidders, bidder 1 and bidder 2, through a FPA or a SPA. The bidders' consumption values, that is, the values for their own consumptions of the item, are independently distributed over the same interval $[c, d]$, with $c < d$, according to absolutely continuous probability measures F_1 and F_2 with density functions f_1 and f_2 that are strictly positive and continuous[8]. I use the same notations F_1 and F_2 for the cumulative distribution functions. As in the standard independent-private-value model, a bidder's consumption value is his own private information.

For the sake of simplicity, I assume that the reserve price is the minimum value c, that participation to the auction is mandatory, and that each bidder has the same probability (1/2) of winning a tie[9].

A resale stage immediately follows the auction[10]. In order to simplify the presentation, I first assume, in this section and in Sections 4, 5, and 6, that the auction winner proposes the resale price. Resale takes place if and

[7] If I applied this selection criterion to my model, I would obtain revenue-equivalence between the SPA and the FPA. Parreiras (2006) obtains the revenue-superiority of the SPA because the signals in his model are strictly affiliated, while here the consumption values are independently distributed.

[8] Many of the results hold true under more general assumptions that allow, for example, density functions that are defined and strictly positive only over $(c, d]$ (as long as they are bounded).

[9] The equilibria remain equilibria under voluntary participation and any tie-breaking rule. The results about the SPA easily extend to an arbitrary binding reserve price.

[10] Equivalently, I assume that the bidders use the same discount factor $\delta = 1$. As long as there is no consumption between auction and resale, all the results translate to the case of arbitrary common discounting by properly discounting consumption values and bids. If, for example, bidders use the same discount factor $\delta < 1$ and resale and consumption occur T periods after the auction rather than immediately following it, the results go through by multiplying all bids by δ^T.

only if the auction loser agrees to this offer. I present the results pertaining to the other bargaining procedure I consider, where the auction loser is the resale-price maker, in the later sections.

An important feature of the model is the information about the auction outcomes that is released after the auction and before resale. Under the full-disclosure regime, the bids are made public after the auction. Under the no-disclosure regime, only the identity of the winner is made public.

With full disclosure, the truth-bidding equilibrium is the only symmetric separating PBE of the SPA. The auction allocation is always efficient in this PBE and resale never occurs. If a bidder increases his bid from his consumption value, the auction outcome changes only when the competing bid lies between his consumption value and his new bid. In this case, the bidder makes a negative payoff at auction and recoups exactly his losses by optimally choosing the other bidder's consumption value, which he knows from observing the losing bid, as the resale price. A bid lower than his consumption value would make him lose at auction and, possibly, buy at resale when he would have won at auction and paid a lower price[11]. Since any symmetric separating equilibrium is efficient, bidders get no payoff from resale and a standard argument shows that the common bidding function cannot be different from the identity function.

I next observe that under full disclosure there exists no asymmetric separating PBE of the SPA or FPA where bidders follow different continuous, strictly increasing, and, for the FPA, differentiable bidding functions. In such a PBE, there would exist a domain of consumption values where a bidder is more aggressive–follows a strictly higher bidding function–than the other bidder. Upwards and downwards deviations by the less aggressive bidder

[11] This is correct as long as the auction winner does not make weakly dominated proposals, that is, prices smaller than his consumption value. If the auction winner offers a price equal to his consumption value, then all bids give the same expected payoffs. The existence, beside the equilibrium bid, of a set of optimal bids is pervasive in PBE's of auctions with resale. Indeed, in Section 6, I use this property to construct behavioral PBE's.

could then not be simultaneously prevented. In fact, if upwards deviations, which bring him no payoff at resale, were not profitable, downwards deviations would be strictly profitable, since they would increase his payoff at resale when the auction allocation is inefficient. In particular, since the same bidding function could not satisfy both bidders' (different) first-order conditions, there exists no pure separating (symmetric or asymmetric) PBE of the FPA with strictly increasing and differentiable bidding functions if $F_1 \neq F_2$.

The lack of separating PBE's as in the two previous paragraphs is due to the signalling bidders may engage in through their bids. Under the no-disclosure regime, signalling is impossible since bids are kept secret and payments are deferred until after resale. As I state in Theorems 1 and 2 below, there then always exist a separating PBE of the FPA and a multiplicity of such PBE's of the SPA. First, I need Definition 1 below.

Definition 1: *Let the (buyer-) virtual-value functions* $v - \frac{1-F_1(v)}{f_1(v)}$ *and* $v - \frac{1-F_2(v)}{f_2(v)}$ *be strictly increasing.*

(i) Then, the (seller's) optimal-resale-price function ρ^s *is the function defined over* $[c,d]^2$ *such that, for all* (w_1, w_2) *in* $[c,d]^2$*, its value at* (w_1, w_2) *is the resale price that maximizes the expected payoff of bidder* k *when his consumption value is* w_k*, he initially owns the item, and bidder* l*'s consumption value is distributed according to the restriction of* F_l *to* $[c, w_l]$*, where* w_k *and* w_l *are the lower and higher components of* (w_1, w_2)*, that is,* $\rho^s(w_1, w_2)$ *is the unique solution to the equation below:*

$$w_k = \rho^s(w_1, w_2) - \frac{F_l(w_l) - F_l(\rho^s(w_1, w_2))}{f_l(\rho^s(w_1, w_2))}, \quad (1)$$

with k *and* l *such that* $\{k, l\} = \{1, 2\}$ *and* $w_k \leq w_l$*.*

(ii) Notation:

$\rho_1^s(v, w) = \rho_2^s(w, v) = \rho^s(v, w)$*, for all* (v, w) *in* $[c,d]^2$*.*

That equation (1) has a unique solution follows easily from the strict monotonicity of the virtual-value functions. The superscript s reminds the reader that $\rho^s(w_1, w_2)$ is the price that is optimal for the seller at resale, that is, the auction winner. According to the notation (ii), ρ_i^s is the function ρ^s with bidder i's consumption value as the first argument.

A strategy of bidder i includes a bidding strategy $\beta_i(.)$–a bidding function, if the strategy is pure–and a resale-offer function $\gamma_i(.;.)$. If bidder i with consumption value v_i follows (β_i, γ_i), he bids $\beta_i(v_i)$ at auction and offers the resale price $\gamma_i(v_i; b_i)$ at resale when he has won the auction with the bid b_i. I focus on couples of strategies $(\beta_1, \gamma_1; \beta_2, \gamma_2)$ that can be completed into PBE's[12]. For the sake of brevity, I also call PBE such a couple of strategies. When β_i is a strictly increasing bidding function, its "extended" inverse β_i^{-1} is the extension of its inverse over bids above the range of β_i, where it takes the constant value d.

I prove in the next section Theorems 1 and 2 below.

Theorem 1: *Let $v - \frac{1-F_1(v)}{f_1(v)}$ and $v - \frac{1-F_2(v)}{f_2(v)}$ be strictly increasing over $[c, d]$. Let φ be a strictly increasing continuous function over $[c, d]$ such that $\varphi(c) = c$ and $\varphi(d) = d$. Let β_1, $\beta_2, \gamma_1, \gamma_2$ be the following bidding and resale-offer functions:*

$$\begin{aligned} \beta_1(v) &= \rho^s(v, \varphi(v)), \\ \beta_2(v) &= \rho^s\left(\varphi^{-1}(v), v\right), \end{aligned}$$

for all v in $[c, d]$;

$$\gamma_i(v; b) = \rho_i^s(v, \max(v, \alpha_j(b))),$$

where α_j is the extended inverse of β_j, $i, j = 1, 2$, $i \neq j$, for all (v, b) in $[c, d] \times [c, +\infty)$. Then, $(\beta_1, \gamma_1; \beta_2, \gamma_2)$ is a PBE of the SPA under no

[12] By adding beliefs and by adding what responses every bidder should give to offers from the other bidder at resale, as functions of the past observed histories.

disclosure. Moreover, the following equalities hold true:

$$\begin{aligned}\alpha_2\beta_1 &= \varphi, (2)\\ \rho^s\left(\alpha_1(b), \alpha_2(b)\right) &= b,\end{aligned}$$

for all v in $[c,d]$ and b in $[\beta_1(c), \beta_1(d)] = [\beta_2(c), \beta_2(d)] = [c,d]$.

From (2) above, φ determines the "intermediate" equilibrium allocation, the allocation after the auction and before resale: bidder 1 with consumption value v_1 wins the auction if and only if, neglecting ties, bidder 2's consumption value v_2 is such that $v_2 < \varphi(v_1)$. If φ is the identity function, the PBE in Theorem 1 is the truth-bidding equilibrium. Since there is an infinity of functions φ as in Theorem 1, the SPA has an infinity of PBE's, even if the bidders are homogeneous, that is, if $F_1 = F_2$. In a PBE as in Theorem 1, a bidder's bid at auction is the resale price at which resale takes place when the other bidder submits the same bid and the tie is broken inefficiently. For example, $\beta_1(v) = \rho^s(v, \varphi(v))$ is the resale price that is proposed and accepted after the tie which bidder 1 wins if $\varphi(v) > v$ and which bidder 2 wins if $\varphi(v) < v$.

Theorem 2: *Let $v - \frac{1-F_1(v)}{f_1(v)}$ and $v - \frac{1-F_2(v)}{f_2(v)}$ be strictly increasing over $[c,d]$. Let β_1, $\beta_2, \gamma_1, \gamma_2$ be the following bidding and resale-offer functions:*

$$\beta_i(v) = \frac{\int_0^{F_i(v)} \rho^s\left(F_1^{-1}(q), F_2^{-1}(q)\right) dq}{F_i(v)} \quad (3)$$

for all v in $[c,d]$;

$$\gamma_i(v;b) = \rho_i^s\left(v, \max\left(v, \alpha_j(b)\right)\right),$$

where α_j is the extended inverse of β_j, $i,j = 1,2$, $i \neq j$, for all (v,b) in $[c,d] \times [c,+\infty)$. Then, $(\beta_1, \gamma_1; \beta_2, \gamma_2)$ is a PBE of the FPA under no *disclosure. The following equality holds true:*

$$\alpha_2\beta_1 = F_2^{-1}F_1.(4)$$

Moreover, β_1, β_2 are the unique bidding functions part of a pure PBE that are, over $(c, d]$, strictly increasing, differentiable, and such that $\beta_1(c) = \beta_2(c) = c$.

From (4) the bidders' equilibrium bid distributions are identical and from (3) a bidder submits the expectation, over all lower bids, of the prices at which resale takes place after both bidders submit the same bid and the tie is broken in favor of the inefficient winner. From Gupta and Lebrun (1999), the equilibrium bidding functions (3) are the same as in the simple model where private information becomes public before resale and the resale price is exogenously determined from the values according to ρ^s. Also from Gupta and Lebrun (1999), the same bid distributions[13] arise at the equilibrium of the symmetric model where both bidders' consumption values are distributed according to G^s such that $(G^s)^{-1}(q) = \rho^s\left(F_1^{-1}(q), F_2^{-1}(q)\right)$, for all q in $[0, 1]$.

Although the PBE Theorem 2 describes is the only pure PBE that satisfies "regularity" conditions, it is not, in general, the unique PBE of the FPA. In fact, I show in Section 6 that, when $F_1 \neq F_2$, there exists a behavioral PBE, where at least one bidder randomizes over a range of bids.

Obtaining the first-order conditions, or FOC's, the PBE's have to satisfy helps gaining intuition for the results. Assume, for example, bidder 1 with consumption value $v = \alpha_1(b)$ slightly increases his bid in the FPA from his equilibrium bid b to $b + db$. If the bid increase does not change the auction allocation, the only first-order effect on bidder 1's expected payoff is the increase db of his payment in case of winning. In fact, since bidder 2 does

[13] Symmetry (of at least some degree) at the equilibrium of asymmetric pure-common-value FPA's was encountered in Wilson (1967), Engelbrecht-Wiggans, Milgrom, and Weber (1983), Hendricks and Porter (1988), Hendricks, Porter and Wilson (1994), Campo, Perrigne, and Vuong (2003), Parreiras (2006), and Cheng and Huang (2007).

not observe bidder 1's bid, the bid increase is without any effect if bidder 1 loses the auction. If bidder 1 wins the auction, the bid increase changes the resale price he proposes at resale. However, since the resale price is optimal, this change has no first-order effect on his expected payoff (by the envelope theorem).

The bid increase changes the auction allocation when bidder 1's bid b is "pivotal," that is, when bidder 2 submits the same bid (up to db) and, consequently, has the consumption value $v_2 = \alpha_2(b)$. Assume first $\alpha_1(b) \leq \alpha_2(b)$. With the initial bid b, bidder 1 loses the auction and there is no profitable resale. With the increased bid $b+db$, bidder 1 wins the auction, pays his bid, and proposes the resale price $\rho^s(\alpha_1(b), \alpha_2(b))$, which is optimal conditional on bidder 2's consumption value belonging to $[c, \alpha_2(b)]$. Because bidder 2's value is actually equal to the upper extremity $\alpha_2(b)$ of this interval, bidder 2 accepts this price and resale occurs. Bidder 1's net gain from increasing his bid is thus $\rho^s(\alpha_1(b), \alpha_2(b)) - b$, the profit from his "intermediation" between the auctioneer and bidder 2. When $\alpha_1(b) \geq \alpha_2(b)$, if he submits b, bidder 1 loses the auction, and accepts the resale price $\rho^s(\alpha_1(b), \alpha_2(b))$, which bidder 2 proposes. If he submits $b + db$, he wins the auction, pays the auction price, and no resale takes place. His net gain from increasing his bid is again $\rho^s(\alpha_1(b), \alpha_2(b)) - b$, here the difference in price he saves by buying at auction rather than at resale.

At equilibrium, the expected total first-order effect of the bid increase should vanish and the necessary FOC for bidder 1 is (5) below,

$$(\rho^s(\alpha_1(b), \alpha_2(b)) - b) \frac{d}{db} F_2(\alpha_2(b))\, db - F_2(\alpha_2(b))\, db = 0 \quad (5)$$

or, equivalently, (7) below with $i = 2$.

The FOC in the SPA can be obtained similarly. However, because a change in a bidder's bid has no effect on the auction price in case of winning, there is no second term as in (5) and the FOC (6) below is the same for both bidders.

FOC in the SPA:

$$\rho^s(\alpha_1(b), \alpha_2(b)) = b. \ (6)$$

FOC's in the FPA:

$$\frac{d}{db}\ln F_i(\alpha_i(b)) = \frac{1}{\rho^s(\alpha_1(b), \alpha_2(b)) - b}, \ i = 1, 2. \ (7)$$

As it can be easily checked, the bidding functions in Theorems 1 and 2 satisfy these FOC's. In the next section, I offer a complete proof of Theorems 1 and 2 by linking my model to a pure-common-value model.

4. Proofs of Theorems 1 and 2

The bidding functions in Theorems 1 and 2 in the previous section are also the equilibrium bidding functions in a model where the bidders have a common value that, as a function of the signals v_1, v_2 they privately observe, is $\rho^s(v_1, v_2)$. Obviously, my model is not equivalent to such a pure-common-value model. It would be if resale took place at the price $\rho^s(v_1, v_2)$ after any inefficient auction allocation; as in Gupta and Lebrun (1999), where the consumption values become public before resale and the resale price is an exogenous function of the consumption values. Then, the bidders' net values for winning the auction would be equal to the resale price. Here however resale occurs under incomplete information and some inefficient auction allocations are not remedied by resale. As a consequence, a bidder's net value for winning the auction is sometimes equal to his consumption value, instead of the resale price.

It turns out that results from the pure-common-value model apply when a few weak conditions are satisfied, among which that the bidders' net values for winning be identical if their bids are identical. This is precisely what happens in my model since, as already encountered in the previous section, resale corrects any inefficient allocation, and the net value is thus common to both bidders, *if* the bidders submit the same bid at auction.

I first obtain, in Lemma 1, the bidders' endogeneous net values for winning at auction and then state, in Lemma 2, and prove the properties that are sufficient for the results from the pure-common-value model to go through. Assume that β_1, β_2 are the bidding functions, strictly increasing and continuous, the bidders are expected to follow. Bidder i's updated beliefs about bidder j's consumption value after winning with a bid b_i are represented by the conditional of F_j on $[c, \alpha_j(b_i)]$. Then, $\gamma_i(v_i; b_i) = \rho_i^s(v_i, \max(v_i, \alpha_j(b_i)))$, as in Theorems 1 and 2, is the smallest resale price not smaller than bidder i's consumption value that maximizes his expected payoff. If the bidders choose their resale prices according to these (optimal) resale-offer functions, their net values are then as in Lemma 1 below.

Lemma 1: *Let $v - \frac{1-F_1(v)}{f_1(v)}$ and $v - \frac{1-F_2(v)}{f_2(v)}$ be strictly increasing over $[c, d]$. Assume bidder i expects bidder j to follow a continuous and strictly increasing bidding function β_j such that $\beta_j(c) = c$, for all $i \neq j$. Assume bidder i offers the resale price according to the optimal resale-offer function γ_i such that $\gamma_i(v; b) = \rho_i^s(v, \max(v, \alpha_j(b)))$, for all $i \neq j$ and all (v, b) in $[c, d] \times [c, +\infty)$. Then, bidder i's net-value function u_i^s, that is, the difference between his utility $u_i^{s,w}$ when winning (gross of the auction price) and his utility $u_i^{s,l}$ when losing is as follows:*

$$\begin{aligned} & u_i^s\left(v_i, v_j; b_i, b_j; \beta_i, \beta_j\right) \\ = \;& \rho_i^s\left(v_i, \max\left(v_i, \alpha_j\left(b_i\right)\right)\right), \text{ if not larger than } v_j; \\ = \;& \rho_j^s\left(v_j, \max\left(v_j, \alpha_i\left(b_j\right)\right)\right), \text{ if not larger than } v_i; \\ = \;& v_i, \text{ otherwise;} \end{aligned}$$

for all couple of bids (b_1, b_2) in $[c, +\infty)^2$, couple of consumption values (v_1, v_2) in $[c, d]^2$, and $i, j = 1, 2$ with $i \neq j$.

Proof: Immediate.

When resale could take place at the price one of the two bidders would offer, bidder i's net value is equal to the resale price. Otherwise, bidder i's net value is, as when resale is forbidden, equal to his consumption value: winning secures him the item, which he could not have obtained at resale. Since it depends, through their inverses, on the bidding functions the bidders are expected to follow, the net values are endogenous.

Because bidder j, $j \neq i$, does not observe b_i when he makes an offer at resale, bidder i's utility $u_i^{s,l}$ when losing does not depend on his bid b_i. Consequently, I may, when looking for equilibria, consider bidder i's net expected utility (gross of the auction price) with respect to the status quo of his losing with probability one. Proceeding this way, bidder i's net expected utility is (8) below:

$$\begin{aligned}
&\int_c^{\alpha_j(b_i)} u_i^{s,w}\left(v_i, v_j; b_i, \beta_j(v_j); \beta_i, \beta_j\right) dF_j(v_j) + \int_{\alpha_j(b_i)}^d u_i^{s,l}\left(v_i, v_j; b_i, \beta_j(v_j); \beta_i, \beta_j\right) dF_j(v_j) \\
&- \int_c^d u_i^{s,l}\left(v_i, v_j; c, \beta_j(v_j); \beta_i, \beta_j\right) dF_j(v_j) \ \ (u_i^{s,l} \text{ independent of } b_i) \\
= &\int_c^{\alpha_j(b_i)} \left(u_i^{s,w}\left(v_i, v_j; b_i, \beta_j(v_j); \beta_i, \beta_j\right) - u_i^{s,l}\left(v_i, v_j; b_i, \beta_j(v_j); \beta_i, \beta_j\right)\right) dF_j(v_j) \\
= &\int_c^{\alpha_j(b_i)} u_i^{s}\left(v_i, v_j; b_i, \beta_j(v_j); \beta_i, \beta_j\right) dF_j(v_j). \ (8)
\end{aligned}$$

Because bidder i's bid b_i can enter his net value only as an argument of his resale price, which, I have assumed, he chooses optimally, $b_i' = b_i$ is a solution of the maximization problem below:

$$b_i \in \arg\max_{b_i' \geq c} \int_c^{\alpha_j(b_i)} u_i^{s}\left(v_i, v_j; b_i', \beta_j(v_j); \beta_i, \beta_j\right) dF_j(v_j). \ (9)$$

By applying to (9) the integral form of (a variant of) the envelope theorem in Milgrom and Segal (2002) to this problem, I prove in Appendix 1 Lemma

2 (i) below, which allows to circumvent the direct dependence of u_i^s on the own bid b_i and to focus on the net value with identical bids. Lemma 2 (i) is the formal expression of the lack of a first-order effect of a bid change on the expected payoff through the resale price.

I also prove in Appendix 1 the rest of Lemma 2. Lemma 2 (ii) comes from the correction by resale of the "worst" possible cases of inefficiency, due to an inefficient allocation after a tie. Lemma 2 (iii) comes from the monotonicity of the reseller's optimal resale price with respect to his consumption value.

Lemma 2: *Under the assumptions of Lemma 1, let u_i^s be bidder i's net-value function as defined in Lemma 1, for all i. Then, for all $i \neq j$:*

(i)–Envelope Result: For all (v_i, b_i) in $[c, d] \times [c, +\infty)$,

$$\begin{aligned} & \int_c^{\alpha_j(b_i)} u_i^s \left(v_i, v_j; b_i, \beta_j(v_j); \beta_i, \beta_j\right) dF_j(v_j) \\ = & \int_c^{\alpha_j(b_i)} u_i^s \left(v_i, v_j; \beta_j(v_j), \beta_j(v_j); \beta_i, \beta_j\right) dF_j(v_j). \end{aligned}$$

(ii)–Common Value for Identical Bids when Bidding as Expected:

For all $b \geq c$,

$$u_1^s(\alpha_1(b), \alpha_2(b); b, b; \beta_1, \beta_2) = u_2^s(\alpha_1(b), \alpha_2(b); b, b; \beta_1, \beta_2) = \rho^s(\alpha_1(b), \alpha_2(b)).$$

(iii)–Monotonicity with respect to Own Type: For all $b \geq c$, $u_i^s\left(v_i, \alpha_j(b); b, b; \beta_i, \beta_j\right)$ is nondecreasing with respect to v_i in $[c, d]$.

Proof: See Appendix 1.

Theorems 1 and 2 then follow easily from the properties in Lemma 2. Indeed, from Lemma 2 (i), bidder i's expected net payoffs (net of the auction price) when his consumption value is v_i and his bid is b are as follows.

Expected Net Payoff in the SPA:

$$\int_c^{\alpha_j(b)} \left(u_i^s\left(v_i, v_j; \beta_j\left(v_j\right), \beta_j\left(v_j\right); \beta_i, \beta_j\right) - \beta_j\left(v_j\right)\right) dF_j\left(v_j\right); \quad (10)$$

Expected Net Payoff in the FPA:

$$\int_c^{\alpha_j(b)} u_i^s\left(v_i, v_j; \beta_j\left(v_j\right), \beta_j\left(v_j\right); \beta_i, \beta_j\right) dF_j\left(v_j\right) - \int_c^{\alpha_j(b)} b dF_j\left(v_j\right). \quad (11)$$

Since, at an equilibrium, b should be optimal if $v_i = \alpha_i(b)$, I obtain from Lemma 2 (ii) the FOC's (6) and (7)[14]. From Gupta and Lebrun (1999), the bidding functions described in Theorem 2 form the unique solution of the two conditions (7)[15]. As in the SPA with pure common value, the single condition (6) admits the multiplicity of solutions described in Theorem 1. From Lemma 2 (iii), the "second-order" condition is satisfied for both auction formats and the first-order conditions are sufficient. Any bidder's expected net payoff is then quasi-concave in the bid and reaches its maximum at the bid the bidding function specifies. Theorems 1 and 2 are proved.

5. Properties of the PBE's

I already mentioned (in Footnote 11) that, in PBE's of auctions with resale, the equilibrium bid is not, in general, a strict best response. Corollary 1 below describes a set of optimal bids of any "equilibrium resale-price taker," that is, a bidder who is less aggressive at his consumption value and

[14] The same FOC's (7) for the FPA would follow from any other choice of optimal resale-offer functions. If a resale-offer function γ_i is optimal, it must satisfy $\gamma_i(\alpha_i(b); b) = \rho_i^s(\alpha_i(b), \alpha_j(b))$ if $\alpha_i(b) < \alpha_j(b)$. If $\alpha_i(b) = \alpha_j(b)$, then, whether resale occurs or not, both bidders' net values for winning are equal to $\alpha_i(b) = \alpha_j(b)$, which is also equal to $\rho_i^s(\alpha_i(b), \alpha_j(b))$.

[15] Together with the immediate boundary condition $\beta_1(d) = \beta_2(d)$.

consequently could not find, after winning the auction, a profitable resale price that would be acceptable to the other bidder. I use Corollary 1 in the next section to construct behavioral PBE's under full disclosure that are equivalent to the PBE's from Theorems 1 and 2 (Section 3).

Consider a small change of bid around b, possibly different from his equilibrium bid, by bidder i with consumption value v. If bidder j with equilibrium bid b proposes, after he wins, a resale price bidder i accepts, the first-order effect of the bid change on bidder i's expected payoff depends only on this resale price and hence is independent of v. Since b is optimal for bidder i with a certain consumption value, possibly different from v, the first-order effect must vanish. By continuity the same holds true for bids in a neighborhood of b and hence bidder i's expected payoff is constant around b. The set of optimal bids in Corollary 1 for bidder i, if he is less aggressive at his consumption value, is precisely the neighborhood of his equilibrium bid made of bids that, if submitted by a winning bidder j, would be followed by offers bidder i would accept.

The function φ^- in Corollary 1 is such that its value at v is the largest point of coincidence between the bidding functions to the left of v. In the SPA, the set in Corollary 1 of optimal bids for a bidder with consumption value v, if he is less aggressive at v, is (from the formulas for β_i in Theorem 1, Section 3) $[\varphi^-(v), v]$. The bidder is indifferent between winning and losing against an opponent who submits a bid b in this set because in both cases, he ends up with the item and pays b, which the opponent would offer after winning. From Theorem 2 (Section 3), the function φ in Corollary 1 is $F_2^{-1}F_1$ for the FPA.

Corollary 1: *Let $(\beta_1, \gamma_1; \beta_2, \gamma_2)$ be a PBE as in Theorem 1 or Theorem 2 and let φ be equal to $\alpha_2\beta_1$. Let φ^- be the function defined over $[c, d]$ as follows:*

$$\varphi^-(v) = \max\{w \in [c, v] \, | \varphi(w) = w\}.$$

Then all bids in

$$\left[\beta_1(\varphi^-(v)), \rho^s(\alpha_1(.), \alpha_2(.))^{-1}(v)\right]$$

are optimal for bidder i with consumption value v in $[c, d]$ if he is less aggressive at v, that is, if $\beta_i(v) < \beta_j(v)$, $j \neq i$[16].

Proof: For example, assume, as in Figure 1 below, that bidder 2 is less aggressive at v, that is, $\beta_1(v) > \beta_2(v)$, or, equivalently, $\varphi(v) > v$ (the proof is similar in the other case). Since bidder 2 with consumption value v wins the auction only if bidder 1's consumption value is smaller than v, no trade can occur after bidder 2 wins. Let b be a bid in $\left(\beta_1(\varphi^-(v)), \rho^s(\alpha_1(.), \alpha_2(.))^{-1}(v)\right)$. Then, $\alpha_1(b) < \alpha_2(b)$ and $\rho^s(\alpha_1(b), \alpha_2(b)) < \min(v, \alpha_2(b))$.

By continuity, there exists a neighborhood of $\alpha_1(b)$ such that, for all v_1 in this neighborhood, $\rho^s(v_1, \alpha_2(\beta_1(v_1)))$ is smaller than v and $\alpha_2(b)$ and is thus, from Lemma 1 (Section 4), equal to both net values $u_2^s(v_1, v; \beta_1(v_1), \beta_1(v_1); \beta_1, \beta_2)$ and $u_2^s(v_1, \alpha_2(b); \beta_1(v_1), \beta_1(v_1); \beta_1, \beta_2)$. Consequently, from (10) and (11) in the previous section, the first-order effect of a bid change from b on bidder 2's expected payoff when his consumption value is v is the same as when his consumption value is $\alpha_2(b)$. From the optimality of b for this later consumption value, this first-order effect vanishes at b. Since this is true for all b in the interval $\left(\beta_1(\varphi^-(v)), \rho^s(\alpha_1(.), \alpha_2(.))^{-1}(v)\right)$, bidder 2's expected payoff must be constant over the closure of this interval. Because it contains the equilibrium bid $\beta_2(v)$, all its elements are optimal bids. ||

FIGURE 1

As stated in Corollary 2 below, the equilibrium resale price characterizes the final, that is, post-resale, equilibrium allocation. In fact, if bidder 1's consumption value is v_1, bidder 2 ends up with the item if and only if his consumption value is above the cut-off $\lambda_\varphi^s(v_1)$, which is the resale price bidder

[16] Or, equivalently, $\varphi(v) < v$, if $i = 1$, and $\varphi(v) > v$, if $i = 2$.

1 offers, if bidder 1 is more aggressive at v_1, and the consumption value at which bidder 2 offers v_1 , if bidder 1 is less aggressive at v_1.

Corollary 2: *Let $(\beta_1, \gamma_1; \beta_2, \gamma_2)$ be a PBE as in Theorem 1 or Theorem 2 and let φ be equal to $\alpha_2\beta_1$. Let λ^s_φ be the function defined over $[c,d]$ as follows:*

$$
\begin{aligned}
\lambda^s_\varphi (v_1) &= \rho^s (v_1, \varphi (v_1)), \text{ if } \varphi (v_1) \geq v_1; \\
\lambda^s_\varphi (v_1) &= \rho^s \left(\varphi^{-1} (.), .\right)^{-1} (v_1), \text{ if } \varphi (v_1) \leq v_1.
\end{aligned}
$$

If bidder 1's and bidder 2's consumption values v_1, v_2 in $[c,d]$ are such that $v_2 <(>)\lambda^s_\varphi (v_1)$, then the equilibrium eventually allocates the item to bidder 1 (2).

Proof: The proof is simple. For example, assume $\varphi (v_1) \geq v_1$. From the definitions of φ and λ^s_φ, $\lambda^s_\varphi (v_1) \leq \varphi (v_1) = \alpha_2\beta_1 (v_1)$. If $v_2 < \lambda^s_\varphi (v_1)$, bidder 2 loses the auction and refuses bidder 1's resale offer. If $\lambda^s_\varphi (v_1) < v_2 < \varphi (v_1)$, bidder 2 loses the auction and accepts bidder 1's resale offer. If $\varphi (v_1) < v_2$, bidder 2 wins the auction and no mutually advantageous resale is possible. ||

6. The Full-Disclosure Regime

6.1 Statement of the Result

I now turn to the full-disclosure regime, where the bids are revealed before resale. A bidding strategy, not necessarily pure, of bidder i is a (regular) conditional probability measure $G_i (.|.)$ with respect to v_i in $[c,d]$. A resale-offer strategy is characterized by a function $\delta_i (.;.)$. If bidder i with consumption value v_i follows $(G_i (.|.), \delta_i)$, he chooses his bid according to $G_i (.|v_i)$ and, after winning the auction where bidder j bids b_j, he offers $\delta_i (v_i; b_j)$ at resale.

Here, the second argument of a resale-offer function δ_i of bidder i is not, as in Section 3, his own bid, but rather bidder j's bid, which he observes.

I specify bidder i's revised beliefs after he observes $b_j \geq c$ through a (regular) conditional probability measure $F_j(.|.)$. That is, the measure $F_j(.|b_j)$ represents the revised beliefs bidder i holds about bidder j's consumption value after he observes bidder j's bid b_j. For short, I call a couple of strategies and beliefs $(G_1(.|.), \delta_1, F_2(.|.); G_2(.|.), \delta_2, F_1(.|.))$ a PBE if it can be completed into one. Corollary 3 below is the main result of this section.

Corollary 3: *Let $\mathcal{E}$ be a PBE of an auction (FPA or SPA) under the no-disclosure regime as in Theorem 1 or Theorem 2 (Section 3). Then, there exists a PBE $\mathcal{E}'$ of the same auction under the full-disclosure regime such that:*

(i) The bid marginal distributions, the interim total expected payoffs, and the final (post-resale) allocation are the same as in $\mathcal{E}$;

(ii) Conditionally on the consumption value of the auction winner, strictly profitable resale takes place with the same probability as in $\mathcal{E}$ and, when this probability is different from zero, at the same price;

(iii) If the auction is the SPA, the auction outcomes–the bids and the allocation before resale–are posterior implemented by $\mathcal{E}'$ or, for short, $\mathcal{E}'$ is posterior implementable;

(iv) There exists a PBE of the auction under no disclosure[17] *with the same bidding strategies and, along the equilibrium path, the same offers from the auction winner as in $\mathcal{E}'$.*

From (i) and (ii), for every PBE of an auction under the no-disclosure regime, there exists a PBE of the same auction under the full-disclosure regime that gives the same expected payoffs and is equivalent to any outside observer. Following Green and Laffont (1987) (see, also, Lopomo 2001), (iii)

[17] (iv) actually holds true for any disclosure policy, including, for example, when "garbled" information about the bids is released.

means that all bids in the support of bidder i's bidding strategy conditional on v_i are optimal for bidder i with consumption value v_i even after he learns bidder j's bid.

If $\mathcal{E}$ is asymmetric, the bidding strategies in $\mathcal{E}'$, although observationally equivalent, are different from those in $\mathcal{E}$. In fact, according to the construction in the next subsection, $\mathcal{E}'$ is not pure in this case: the less aggressive bidder, who is the equilibrium resale-price taker, randomizes over a range of bids. From (iv), $\mathcal{E}'$ then also gives a PBE, different from $\mathcal{E}$, of the auction under no disclosure.

6.2 Proof of the Result

Let $\mathcal{E} = (\beta_1, \gamma_1; \beta_2, \gamma_2)$ be a PBE as in Theorem 1 or Theorem 2 (Section 3). In order to prove Corollary 3, I first construct a couple $\mathcal{E}' = (G_1(.|.), \delta_1, F_2(.|.); G_2(.|.), \delta_2, F_1(.|.))$ of strategies and (consistent) revised beliefs as in Lemma 3 below.

The construction proceeds by having any bidder who bids less aggressively in $\mathcal{E}$ mix over his set of optimal bids (Corollary 1 in Section 5 describes) in such a way that when the other bidder wins and observes the losing bid, he chooses the same resale-price as in $\mathcal{E}$. The differential equation this requirement implies determines the auction winner's beliefs conditional on the losing bid. The marginal bid distribution is taken to be the same as in $\mathcal{E}$. These conditional and marginal distributions then determine a joint distribution. I prove that its conditional distribution with respect to the value is a bidding strategy of the less aggressive bidder by proving, using Milgrom and Weber (1985)'s terminology, that the joint distribution is a "distributional strategy," that is, that its marginal value distribution is the bidder's actual value distribution. In the statement of Lemma 3, as throughout the text, bidder i is less aggressive at v_i in $\mathcal{E}$ if and only if $\beta_i(v_i) < \beta_j(v_i)$[18].

[18] That is, $\varphi(v_1) < v_1$ if $i = 1$ and $\varphi(v_2) > v_2$ if $i = 2$, where $\varphi = \alpha_2\beta_1$.

Lemma 3: *Let $\mathcal{E} = (\beta_1, \gamma_1; \beta_2, \gamma_2)$ be a PBE of an auction (FPA or SPA) under the no-disclosure regime as in Theorem 1 or Theorem 2 (Section 3). Then, the four-step construction below produces a couple $\mathcal{E}' = (G_1(.|.), \delta_1, F_2(.|.); G_2(.|.), \delta_2, F_1(.|.))$ of bidding and resale-offer strategies and consistent revised beliefs.*

Step 1. Construction of the bid supports: If bidder i is less aggressive at v_i in $\mathcal{E}$, the support of $G_i(.|v_i)$ is the interval of optimal bids described in Corollary 1 (Section 5). Otherwise, the support of $G_i(.|v_i)$ is $\{\beta_i(v_i)\}$.

Step 2. Construction of revised beliefs $F_1(.|.)$ and $F_2(.|.)$ that are consistent with the supports in Step 1 and such that, when mutually advantageous resale is possible along the equilibrium path of $\mathcal{E}$, the auction winner finds optimal to offer the same resale prices as in $\mathcal{E}$.

Step 3. Construction of the bidding strategy $G_i(.|.)$ as the conditional distribution of the bid with respect to the consumption value from the joint distribution of the consumption-value-bid couples generated by the marginal $F_i\alpha_i$ of bidder i's bid in $\mathcal{E}$ and the conditional $F_i(.|.)$ from Step 2, for all $i = 1, 2$.

Step 4. Extension of the construction of optimal resale-offer functions from the domains in Step 2, where resale is possible, to the whole definition domain $[c, d] \times [c, +\infty)$ such that, when the auction winner has followed the bidding strategy in Step 3 and has the revised beliefs from Step 2, the resale offer does not depend on the bid from the auction loser, as follows:

$$\begin{aligned} \delta_i(v_i; b_j) &= \max\left(\rho_i^s(v_i, \alpha_j \beta_i(v_i)), v_i\right), \text{ if } b_j \leq \beta_i(v_i); \\ &= \max\left\{\min\left(\rho^s(\alpha_1(b_j), \alpha_2(b_j)), \alpha_j(b_j)\right), v_i\right\}, \text{ if } b_j > \beta_i(v_i). \end{aligned}$$

Proof: See Appendix 2.

The proof that $\mathcal{E}'$ as constructed in the lemma above is actually a PBE is similar to the proof, in Section 4, of Theorems 1 and 2. I first define, in

Lemma 4, endogeneous net values and then obtain, in Lemma 5, properties that extend properties from the pure-common-value model and are sufficient for equilibrium. Since I only consider β_i, β_j from a given $\mathcal{E}$, I drop them from the argument of the net values u_i^s.

Lemma 4: *Let $v - \frac{1-F_1(v)}{f_1(v)}$ and $v - \frac{1-F_2(v)}{f_2(v)}$ be strictly increasing over $[c, d]$. Let $\mathcal{E} = (\beta_1, \gamma_1; \beta_2, \gamma_2)$ be a PBE of an auction (FPA or SPA) under the no-disclosure regime as in Theorem 1 or Theorem 2 (Section 3) and let $\mathcal{E}' = (G_1(.|.), \delta_1, F_2(.|.); G_2(.|.), \delta_2, F_1(.|.))$ be the couple of strategies and beliefs under the full-disclosure regime that is constructed from $\mathcal{E}$ as in Lemma 3. Assume bidder i expects bidder j to follow $(G_j(.|.), \delta_j)$, for all $i \neq j$. Assume further that bidder i revises his beliefs according to $F_i(.|.)$ and offers the resale price according to the optimal resale-offer function δ_i, for all $i = 1, 2$. Then, bidder i's net-value function u_i^s, that is, the difference between his utility $u_i^{s,w}$ when winning and his utility $u_i^{s,l}$ when losing is as follows:*

$$
\begin{aligned}
& u_i^s(v_i, v_j; b_i, b_j) \\
= \; & \delta_i(v_i; b_j), \text{ if not larger than } v_j; \\
= \; & \delta_j(v_j; b_i), \text{ if not larger than } v_i; \\
= \; & v_i, \text{ otherwise;}
\end{aligned}
$$

for all couple of bids (b_1, b_2) in $[c, +\infty)^2$, couple of consumption values (v_1, v_2) in $[c, d]^2$, and $i, j = 1, 2$ with $i \neq j$.

Proof: Immediate.

Here, contrary to Section 4, bidder i's utility in case of winning $u_i^{s,w}$ depends on bidder j's bid b_j, which bidder i observes after winning, and is independent of bidder i's own bid b_i. Thus, I may consider bidder i's winning with probability one as his status quo. His net expected utility is then:

$$
\begin{aligned}
& \int_c^{b_i} \int u_i^{s,w}\left(v_i, v_j; b_i, b_j\right) dF_j\left(v_j|b_j\right) dG_j\left(b_j\right) \\
& + \int_{b_i}^{d} \int u_i^{s,l}\left(v_i, v_j; b_i, b_j\right) dF_j\left(v_j|b_j\right) dG_j\left(b_j\right) \\
& - \int_c^{d} \int u_i^{s,w}\left(v_i, v_j; d, b_j\right) dF_j\left(v_j|b_j\right) dG_j\left(b_j\right) \;\; \left(u_i^{s,w} \text{ independent of } b_i\right) \\
= & \int_d^{b_i} \int \left(u_i^{s,w}\left(v_i, v_j; b_i, b_j\right) - u_i^{s,l}\left(v_i, v_j; b_i, b_j\right)\right) dF_j\left(v_j|b_j\right) dG_j\left(b_j\right),
\end{aligned}
$$

where I denote G_j bidder j's marginal bid probability distribution in $\mathcal{E}'$, which, from Step 3 of the construction in Lemma 3, is equal to the marginal distribution $F_j\alpha_j$ in $\mathcal{E}$.

Lemma 5 below is similar to Lemma 2 (Section 4) and follows from the construction of $\mathcal{E}'$. Lemma 5 (i.1) and (i.2) come from the independence, by Step 4 of the construction in Lemma 3, of the resale price on the auction loser's bid, if the auction winner follows his bidding strategy in $\mathcal{E}'$. Lemma 5 (ii) holds true because, although the bidders use behavioral bidding strategies, resale remedies in $\mathcal{E}'$ any inefficient outcome after a tie. Lemma 5 (iii) follows from the monotonicity of a bidder's optimal expected payoff at resale with respect to his consumption value.

Lemma 5: *Under the assumptions of Lemma 3, let u_i^s be bidder i's net-value function as defined in Lemma 4, for all i. Then:*

(i.1) For all (v_i, b_i) in $[c,d] \times [c,+\infty)$ and all $b_j \geq b_i$:

$$
\begin{aligned}
& \int u_i^{s}\left(v_i, v_j; b_i, b_j\right) dF_j\left(v_j|b_j\right) \\
= & \int u_i^{s}\left(v_i, v_j; b_j, b_j\right) dF_j\left(v_j|b_j\right)
\end{aligned}
$$

(i.2) For all (v_i, b_i) *in* $[c,d] \times [c,+\infty)$*:*

$$\begin{aligned} & \int_d^{b_i} \int u_i^s (v_i, v_j; b_i, b_j)\, dF_j (v_j|b_j)\, dF_j\alpha_j (b_j) \\ = & \int_d^{b_i} \int u_i^s (v_i, v_j; b_j, b_j)\, dF_j (v_j|b_j)\, dF_j\alpha_j (b_j) . \end{aligned}$$

(ii) For all $b \geq c$*:*

$$\int u_i^s (\alpha_i (b), v_j; b, b)\, dF_j (v_j|b) = \rho^s (\alpha_1 (b), \alpha_2 (b)) .$$

(iii) For all $b \geq c$, $\int u_i^s (v_i, v_j; b, b)\, dF_j (v_j|b)$ *is nondecreasing with respect to* v_i *in* $[c, d]$.

Proof: See Appendix 2.

As I show in Appendix 2, Corollary 3 can be proved straightforwardly from the properties in Lemma 5. Proceeding as in the proofs in Section 4 of Theorems 1 and 2 and the proof of Corollary 1 (Section 5), one finds that bidder i's equilibrium bid in $\mathcal{E}$ and his optimal bids Corollary 1 describes remain optimal when bidder j follows his strategy in $\mathcal{E}'$ and there is full disclosure. Since the sets of these bids coincide with the ranges of bids in $\mathcal{E}'$, $\mathcal{E}'$ is a PBE. By construction, the marginal bid distributions and the resale prices the bidders offer are the same in $\mathcal{E}'$ as in $\mathcal{E}$. Appealing to Myerson (1981)'s general equivalence result, I then prove the equivalence between $\mathcal{E}'$ and $\mathcal{E}$ from the equality between their final allocations (which are both determined by the function λ_φ^s).

I should emphasize that, although the final allocations in $\mathcal{E}'$ and $\mathcal{E}$ are identical, the intermediate allocations, after the auction and before resale, differ with strictly positive probability if the equilibria are asymmetric. For example, in Figure 1 (Section 5), if bidder 2's consumption value is w and bidder 1's is $\alpha_1(b_2)$, bidder 1 wins the auction in $\mathcal{E}$ with probability one and loses it with strictly positive probability in $\mathcal{E}'$.

7. The Other Bargaining Procedure at Resale

The methods above can also be applied to auctions after which the auction loser sets the price at resale. The main differences are that the common net value with identical bids is now the buyer's optimal resale price ρ^b, defined below, and that an equivalent PBE of an auction under full disclosure is constructed from a given PBE of the auction under no disclosure by having the more aggressive bidder, who is now the equilibrium resale-price taker, use a behavioral bidding strategy.

Definition 2: *Let the (seller-) virtual-value functions* $v + \frac{F_1(v)}{f_1(v)}$ *and* $v + \frac{F_2(v)}{f_2(v)}$ *be strictly increasing.*

(i) Then, the (buyer's) optimal-resale-price function ρ^b *is the function defined over* $[c,d]^2$ *such that, for all* (w_1, w_2) *in* $[c,d]^2$*, its value at* (w_1, w_2) *is the resale price that maximizes bidder* l*'s expected payoff when his consumption value is* w_l *and the consumption value of bidder* k*, who initially owns the item, is distributed according to the restriction of* F_k *to* $[w_k, d]$*, where* w_k *and* w_l *are the lower and higher components of* (w_1, w_2)*, that is,* $\rho^b(w_1, w_2)$ *is the unique solution to the equation below:*

$$w_l = \rho^b(w_1, w_2) - \frac{F_k(w_k) - F_k(\rho^b(w_1, w_2))}{f_k(\rho^b(w_1, w_2))},$$

with k *and* l *such that* $\{k, l\} = \{1, 2\}$ *and* $w_k \leq w_l$*.*

(ii) Notation:

$$\rho_1^b(v, w) = \rho_2^b(w, v) = \rho^b(v, w), \text{ for all } (v, w) \text{ in } [c,d]^2.$$

The function over $[c,d]$ that determines the final allocation is here λ_φ^b

defined as follows:

$$\begin{aligned}\lambda_{\varphi}^{b}(v_1) &= \rho^{b}(v_1, \varphi(v_1)), \text{ if } \varphi(v_1) \leq v_1;\\ \lambda_{\varphi}^{b}(v_1) &= \rho^{b}\left(\varphi^{-1}(.), .\right)^{-1}(v_1), \text{ if } \varphi(v_1) \geq v_1,\end{aligned}$$

for all v_1 in $[c, d]$, if φ is a strictly increasing continuous function such that $\varphi(c) = c$ and $\varphi(d) = d$.

In Appendix 3, I state Theorems A1, A2, and Corollaries A1, A2, A3, which are for this new bargaining procedure what Theorems 1, 2 (Section 3) and Corollaries 1, 2 (Section 5), 3 (Section 6) were for the previous bargaining procedure. I omit the proofs since they are similar to the previous proofs.

8. Comparative Statics

In this section, I study the impact on the auctioneer's expected revenues of a change of the bargaining procedure or the auction format. The intuition, gained from models with resale under complete information (see Gupta and Lebrun 1999 and Haile 2003), according to which more bargaining power to the reseller should imply, through a higher option value, more aggressive bidding can mislead here. In fact, from the results in the previous sections, bidding behavior is determined by the bidders' common net value–the resale price $\rho^{x}(v_1, v_2)$, with $x = s$ or b–when they submit the same bid. For the FPA, since the bids are identically distributed at the equilibrium, only the values $\rho^{x}\left(F_1^{-1}(q), F_2^{-1}(q)\right)$ (the values $(G^{x})^{-1}(q)$ of the quantile function in Section 3), for q in $[0, 1]$, matter. Contrary to resale under complete information, there exist, as I show in Appendix 4, examples where, for some q, the resale price $\rho^{s}\left(F_1^{-1}(q), F_2^{-1}(q)\right)$ the auction winner offers is smaller than the resale price $\rho^{b}\left(F_1^{-1}(q), F_2^{-1}(q)\right)$ the auction loser offers and such that the equilibrium bidding functions in the FPA under no disclosure are smaller, for some consumption values, when the auction winner sets the price. In the equivalent PBE's under full disclosure, the equilibrium bid distributions then are not stochastically larger when the auction winner has the whole bargaining power.

Although the equilibrium bid distributions do not generally dominate those with the auction loser as the resale-price maker, I now establish that the auctioneer's expected revenues are higher with the auction winner as the resale-price maker in the FPA. The main tool in the proof is the revenue equivalence theorem in Myerson (1981)'s general form, I already used in Section 6. Since in all the PBE's in this paper any bidder with the smallest consumption value c receives a payoff of zero, Myerson (1981) implies that the auctioneer's expected revenues are determined by the final allocation of the item. Moreover, from Myerson (1981) again, in equilibrium, a couple of consumption values (v_1, v_2) contributes to the auctioneer's expected revenues the (buyer) virtual consumption value of the eventual owner. The following definitions are convenient.

Definition 3:

(i) For all $i = 1, 2$, the buyer-virtual-value function ω_i^b and the seller-virtual-value function ω_i^s are defined over $[c, d]$ as follows:

$$\omega_i^b(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}; \omega_i^s(v_i) = v_i + \frac{F_i(v_i)}{f_i(v_i)}.$$

(ii) Assume ω_1^b, ω_2^b are strictly increasing and $\omega_1^b(c) \geq \omega_2^b(c)$. The optimal-allocation function ψ is the function $\left(\omega_2^b\right)^{-1} \omega_1^b$, that is, it is the unique function that satisfies (12) below:

$$\omega_2^b(\psi(v)) = \omega_1^b(v), \quad (12)$$

for all v in $[c, d]$.

Assuming, as in (ii) above, that bidder 1 has the higher (buyer) virtual value when both bidders consumption values are c is without loss of generality and is equivalent to $f_1(c) \geq f_2(c)$. The function ψ in (ii) defines Myerson (1981)'s optimal allocation: any incentive-compatible mechanism that allocates the item to bidder 1 if $v_1 > \psi(v_2)$, to bidder 2 otherwise, and that leaves no payoff to any bidder with consumption value c maximizes the auctioneer's revenues among the mechanisms that always sell the item. In Lemma 6 below, λ^s_φ, where $\varphi = F_2^{-1}F_1$, is the function that determines the final equilibrium allocation, defined in Corollary 2 (Section 5), of the FPA where the auction winner sets the resale price. When the auction loser sets the resale price, the final equilibrium allocation is determined by the function λ^b_φ, defined in the previous section.

Lemma 6: *Let $\omega^s_1, \omega^s_2, \omega^b_1, \omega^b_2$ be strictly increasing and $\omega^b_1(c) \geq \omega^b_2(c)$. Let φ be equal to $F_2^{-1}F_1$. For all v in $[c, d]$, if $\varphi(v) = v$, then $\lambda^s_\varphi(v) = \lambda^b_\varphi(v) = v$ and, if $\varphi(v) \neq v$, then:*

$$\lambda^s_\varphi(v) > (<)\lambda^b_\varphi(v) \text{ if and only if } \lambda^s_\varphi(v) < (>)\psi(v).$$

Proof: See Appendix 5.

From Lemma 6, wherever the two final-allocation functions λ^b_φ and λ^s_φ differ, λ^s_φ is closer to the optimal-allocation function ψ and thus makes the better choice by choosing the bidder with the higher virtual consumption value. From Myerson (1981), higher expected revenues accrue to the auctioneer under λ^s_φ and Corollary 4 below follows.

Corollary 4: *Let $\omega^s_1, \omega^s_2, \omega^b_1, \omega^b_2$ be strictly increasing. Let R^s be the auctioneer's equilibrium expected revenues from the FPA where the auction winner is the price maker at resale and R^b when the auction loser is the price maker. Then:*

$$R^s \geq R^b.$$

Assume the auction winner is the resale-price maker. If there exists a nondecreasing function φ^* such that $\lambda^s_{\varphi^*} = \psi$, then the PBE of the SPA constructed from a strictly increasing and continuous function φ, with $\varphi(c) = c$ and $\varphi(d) = d$, that is close to φ^* gives revenues close to the optimal revenues since its final allocation λ^s_φ is close to ψ. Assumption A^s below, which extends Zheng (2002)'s "Resale Monotonicity Assumption" to unranked hazard rates in the two-bidder case, is adapted from Lebrun (2007) and guarantees the existence of such a function φ^*. According to Assumption A^s, when the optimal allocation is biased in favor of bidder 1, for example, for some consumption value v, there exists an intermediate allocation that is further biased in favor of this bidder and is defined by a nondecreasing function such that, after receiving the item at the intermediate stage, he offers $\psi(v)$ as the resale-price.

Assumption A^s: Let ω^b_1, ω^b_2 be strictly increasing and $\omega^b_1(c) \geq \omega^b_2(c)$.

(i) The unique continuous function μ^s_2 defined over $C = \{v \in [c,d] \,|\psi(v) \geq v\}$ and such that $\mu^s_2(v) \geq \psi(v)$ and $\rho^s(v, \mu^s_2(v)) = \psi(v)$, for all v in C, is nondecreasing.

(ii) The unique continuous function μ^s_1 defined over $D = \{v \in [c,d] \,|\psi(v) \leq v\}$ and such that $\mu^s_1(v) \geq \psi^{-1}(v)$ and $\rho^s(\mu^s_1(v), v) = \psi^{-1}(v)$, for all v in D, is nondecreasing.

The existence of the functions μ^s_1, μ^s_2 as defined above comes from the continuity and strict monotonicity of ρ^s. The function φ^* can be constructed as follows: $\varphi^* = \mu^s_2$ over C and $\varphi^* = (\mu^s_1)^{-1}$ over $\mu_1(D)$. Corollary 5 below follows.

Corollary 5: *Let the auction winner be the resale-price maker and let Assumption A^s be satisfied. Then, for any incentive-compatible and individually rational mechanism that sells the item with probability one, there exist PBE's of the SPA that give either higher or arbitrarily close revenues.*

Assumption A^b, again adapted from Lebrun (2007), and Corollary 6 below are relevant to the other bargaining procedure at resale[19].

Assumption A^b: Let $\omega_1^s, \omega_2^s, \omega_1^b, \omega_2^b$ be strictly increasing and $f_1(c) = f_2(c)$.

(i) The unique continuous function μ_1^b defined over C and such that $\mu_1^b(v) \leq \psi^{-1}(v)$ and $\rho^b\left(\mu_1^b(v), v\right) = \psi^{-1}(v)$, for all v in C, is nondecreasing.

(ii) The unique continuous function μ_2^b defined over D and such that $\mu_2^b(v) \leq \psi(v)$ and $\rho^b\left(v, \mu_2^b(v)\right) = \psi(v)$, for all v in D, is nondecreasing.

Corollary 6: *Let the auction loser be the resale-price maker and let Assumption A^b be satisfied. Then, for any incentive-compatible and individually rational mechanism that sells the item with probability one, there exist PBE's of the SPA that give either higher or arbitrarily close revenues.*

The PBE of the SPA that is constructed from $\varphi = F_2^{-1}F_1$ allocates the item as the PBE of the FPA does and hence brings the same expected revenues. In the symmetric case $F_1 = F_2$, the functions λ_φ^s, λ_φ^b and ψ are all equal to the identity function and the PBE of the FPA is an optimal mechanism. As stated in Lemma 7 below, this is no longer the case in the asymmetric case $F_1 \neq F_2$. This point is most clearly proved under the assumption of differentiability of ψ.

Lemma 7: *Let $\omega_1^s, \omega_2^s, \omega_1^b, \omega_2^b$ be strictly increasing and $\omega_1^b(c) \geq \omega_2^b(c)$. Let ψ be differentiable. If $\varphi = F_2^{-1}F_1$ and if F_1 is not everywhere equal to F_2, then neither λ_φ^s nor λ_φ^b is everywhere equal to ψ.*

[19] If $f_1(c) > f_2(c)$, then $\psi(c) > c$ and there exists no φ^* such that $\lambda_{\varphi^*}^b = \psi$. In fact, if there existed such a function φ^*, one would have $c \leq (\varphi^*)^{-1}(v) = \mu_1^b(v) \leq \psi^{-1}(v)$, for all v in $E = \{v \in [\psi(c), d] \,|\psi(v) \geq v\}$, and thus $\varphi^*(c) = \psi(c) > c$. Consequently, bidder 2 with consumption value $\psi(c) > c$ would choose, as a buyer at resale, the price c when he believes that bidder 1's consumption value is distributed over $[c, d]$. Since this resale price would never be accepted, it would clearly not be optimal. In Assumption A^b, I rule out the inequality $f_1(c) > f_2(c)$ by assuming rather $f_1(c) = f_2(c)$. Note that φ^* should then be defined as $\left(\mu_1^b\right)^{-1}$ over C and μ_2^b over D.

Proof: See Appendix 5.

From Lemma 7, if the bidders are heterogeneous, that is, if $F_1 \neq F_2$, there exists an interval where λ_{φ}^{x}, with $\varphi = F_2^{-1} F_1$, is everywhere different from ψ, for all $x = s, b$. By slightly moving φ over this interval, while keeping it continuous and strictly increasing, towards and away from ψ, one makes λ_{φ}^{x} move in the same direction and Corollary 7 below follows.

Corollary 7: *Let $\omega_1^s, \omega_2^s, \omega_1^b, \omega_2^b$ be strictly increasing and $\omega_1^b(c) \geq \omega_2^b(c)$. Let ψ be differentiable. Then:*

(i) When $F_1 = F_2$, the PBE of the FPA gives revenues equal to the maximum revenues from PBE's of the SPA.

(ii) When $F_1 \neq F_2$, the revenues from the PBE of the FPA are strictly smaller than the revenues from some PBE's of the SPA and strictly larger than the revenues from some others.

Lemma 7, combined with Lemma 6, also implies that the inequality between the expected revenues from the FPA under different bargaining procedures at resale in Corollary 4 is strict when the bidders are heterogeneous. Obviously, when the bidders are homogeneous, the PBE of the FPA is efficient, no resale occurs, and a change of the resale-price maker has no effect.

9. (k_1, k_2)-Price Auctions.

In Appendix 6, I show that the PBE's of the SPA in the previous sections are robust to changes in the value distribution of the equilibrium resale-price maker–the more aggressive bidder, given his consumption value, when the auction winner sets the price and the less aggressive bidder when the auction loser sets the resale price. Using the same methods as in the previous sections, I also obtain a class of PBE's where bidders use discontinuous bidding functions or bidding strategies with nonconvex supports. In the same appendix, I show the relation between Garratt and Tröger (2006) and Garratt, Tröger, and Zheng (2006)'s equilibria without discounting and some of these PBE's. In this section, I present another extension of the results: to a class of hybrid auctions.

For k in $(0,1)$, Güth and van Damme (1986) define the k-price auction, or k-PA, as the auction where the highest bidder wins and pays the weighted average of his bid and the second highest bid with respective weights k and $1-k$. If $k=0$, the k-PA is the SPA and if $k=1$ it is the FPA. I further extend the definition to allow discriminatory auction rules. In a (k_1,k_2)-price auction, or (k_1,k_2)-PA, the auction price is computed with the weights k_1 and $1-k_1$ when bidder 1 wins and k_2 and $1-k_2$ when bidder 2 wins.

The analysis of the FPA easily extends to such (k_1,k_2)-PA's, where k_1,k_2 belong to $(0,1]$. For example, under the no-disclosure regime, the FOC (7) becomes (13) below, where $x=s$ or b:

$$\frac{d}{db}\ln F_i\left(\alpha_i\left(b\right)\right)^{1/k_j}=\frac{1}{\rho^x\left(\alpha_1\left(b\right),\alpha_2\left(b\right)\right)-b},\ i,j=1,2,\ \neq j,\ (13)$$

which[20] implies $F_1^{1/k_2}\left(\alpha_1\right)=F_2^{1/k_1}\left(\alpha_2\right)$, or, equivalently, (14) in Corollary 8 (i) below. The equilibrium bidding functions are then as in (15).

From (14), the function φ that determines the post-auction allocation and hence the function λ_φ^x that determines the final allocation depend only on the ratio k_1/k_2. Consequently, all k-PA's, where $k_1=k_2=k$, give the same payoffs as the FPA. As another particular consequence, for all ς in $(0,1)$, the (k_1,k_2)-PA and the $(\varsigma k_1,\varsigma k_2)$-PA give the same payoffs to the bidders and the auctioneer as the SPA where the bidders follow the PBE that is constructed from the function $\varphi=F_2^{-1}F_1^{k_1/k_2}$. In Appendix 7, I prove that, if ζ tends towards zero, the PBE of the $(\varsigma k_1,\varsigma k_2)$-PA tends towards this payoff-equivalent PBE of the SPA. Corollary 8 below follows.

[20] With the boundary condition $\beta_1\left(d\right)=\beta_2\left(d\right)$.

The construction of a PBE under full disclosure that is equivalent to the PBE under no disclosure proceeds as for the FPA.

Corollary 8: *Let* k_1, k_2 *be in* $(0,1]$.

(i) Theorem 2 and Corollary 3, when the auction winner is the resale-price maker, and Theorem A2 and Corollary A3, when the auction loser is the resale-price maker, apply to the (k_1, k_2)*-PA if (4) and (A3.2) are replaced by (14) below, (3) is replaced by (15) below with* $x = s$*, and (A3.1) is replaced by (15) with* $x = b$*:*

$$\begin{aligned}\alpha_2\beta_1(v) &= F_2^{-1}\left(F_1(v)^{k_1/k_2}\right), \quad (14)\\ \beta_i(v) &= \frac{\int_0^{F_i(v)^{1/k_j}} \rho^x\left(F_1^{-1}\left(q^{k_2}\right), F_2^{-1}\left(q^{k_1}\right)\right) dq}{F_i(v)^{1/k_j}}, \; i, j = 1, 2, \; i \neq j. \quad (15)\end{aligned}$$

(ii) For all ζ *in* $(0,1)$*, the PBE of the* $(\varsigma k_1, \varsigma k_2)$*-PA is payoff equivalent to the PBE that is defined as in Theorems 1 and A1 from* $\varphi = F_2^{-1} F_1^{k_1/k_2}$ *in the SPA. When* ς *tends towards zero, the PBE of the* $(\varsigma k_1, \varsigma k_2)$*-PA tends towards this payoff-equivalent PBE of the SPA.*

Proof: See Appendix 7.

From Corollary 8, any PBE of the SPA that is constructed from a function $\varphi = F_2^{-1} F_1^l$, where $l > 0$, is the limit of unique (regular) PBE's of auctions that are arbitrarily "close" to the SPA, that is, where an arbitrarily small part of the auction winner's payment is proportional to his bid.

10. Conclusion

In my model of auctions with resale, although the consumption values are independently distributed and resale occurs under incomplete information, the endogeneous net values are equal across bidders who submit the same bid.

From this property and two others, I was able to use the methods developed for the pure-common-value model and I characterized an infinity of equilibria of the SPA and a unique equilibrium of the FPA. For every equilibrium of either auction with no bid disclosure, I constructed an equivalent equilibrium of the same auction with full bid disclosure. All these equilibria of the SPA with full bid disclosure satisfy a no-regret property: after learning the bids, no bidder regrets his own. Furthermore, all asymmetric equilibria of the SPA are somewhat robust to changes in the value distributions.

I showed an equilibrium of the SPA that gives the same expected revenues as the equilibrium of the FPA. With heterogeneous bidders, I showed equilibria of the SPA that give strictly higher expected revenues. For the FPA, I proved that the expected revenues are higher when the price setter at resale is the auction winner. Under some assumptions, I established that the range of revenues from the SPA does not depend on the bargaining procedure.

I then extended the results to (k_1, k_2)-PA's, a class of auctions that includes the SPA, the FPA, and the k-PA's. I proved that as the (k_1, k_2)-PA tends towards the SPA, with a constant ratio k_1/k_2, its equilibrium tends towards one of the equilibria of the SPA.

Appendix 1

Lemma A1: *Let $v - \frac{1-F_1(v)}{f_1(v)}$ and $v - \frac{1-F_2(v)}{f_2(v)}$ be strictly increasing over $[c,d]$. Let β_1, β_2 be bidding functions and u_1^s, u_2^s be net-value functions as in Lemma 1 (Section 4). Let $i, j = 1, 2$ be such that $i \neq j$. Then, for all $b_i \geq c$ and v_i in $[c,d]$, the function $\widetilde{u}_i^s(v_j) = u_i^s\left(v_i, v_j; b_i, \beta_j(v_j); \beta_i, \beta_j\right)$ of v_j is continuous at $v_j = \alpha_j(b_i)$ and almost all other v_j in $[c,d]$.*

Proof: From the definition of u_i^s in Lemma 1 and the continuity of ρ^s and α_i, $\widetilde{u}_i^s$ is continuous at v_j if $v_j \neq \rho_i^s(v_i, \max(v_i, \alpha_j(b_i)))$. Assume v_j in $[c,d]$ is such that $v_j = \alpha_j(b_i)$ and $v_j = \rho_i^s(v_i, \max(v_i, \alpha_j(b_i)))$. Then,

$v_j = v_i$. Since the function $\widetilde{u}_i^s$ always lies between v_i and v_j, it is continuous if $v_j = v_i$ and Lemma A1 follows. ||

Proof of Lemma 2:

Proof of (i): Through the change of variables $w_j = \alpha_j(b_i)$, (9) implies

$$w_j \in \arg\max_{w'_j \in [c,d]} \int_c^{w_j} u_i^s\left(v_i, v_j; \beta_j\left(w'_j\right), \beta_j(v_j); \beta_i, \beta_j\right) dF_j(v_j), \quad \text{(A1.1)}$$

for all w_j in $[c, d]$. For all w'_j in $[c, d]$, the objective function in (A1.1), as an integral, is absolutely continuous with respect to w_j. From Lemma A1 and the continuity of f_j, the integrand is continuous with respect to v_j almost everywhere in $[c, d]$. Consequently, the derivative of the objective function at w_j exists and is equal to $u_i^x\left(v_i, w_j; \beta_j\left(w'_j\right), \beta_j(w_j); \beta_i, \beta_j\right) f_j(w_j)$, for almost all w_j in $[c, d]$. Since u_i^s and f_j are bounded, the assumptions of a variant[21] of Theorem 2 in Milgrom and Segal (2002) are satisfied. From this variant and the change of variables $w_j = \alpha_j(b_i)$, (i) follows for all b_i in $\left[c, \beta_j(d)\right]$.

(9) implies that the objective function is constant with respect to b'_i and b_i in $\left[\beta_j(d), +\infty\right)$. (i) then follows.

Proof of (ii): If $\alpha_1(b) = \alpha_2(b)$, Lemma 1 implies $u_i^s(\alpha_1(b), \alpha_2(b); b, b; \beta_1, \beta_2) = \alpha_i(b)$. Since $\rho^s(\alpha_1(b), \alpha_2(b)) = \alpha_i(b)$, (ii) follows. If $\alpha_1(b) \neq \alpha_2(b)$, $\min(\alpha_1(b), \alpha_2(b)) < \rho^s(\alpha_1(b), \alpha_2(b)) < \max(\alpha_1(b), \alpha_2(b))$ and (ii) follows from Lemma 1.

Proof of (iii): $u_i^s\left(v_i, \alpha_j(b); b, b; \beta_i, \beta_j\right)$ is, from Lemma 1, equal to $\rho_i^s(v_i, \alpha_j(b))$ if $v_i < \alpha_j(b)$, to $\alpha_j(b)$ if $v_i = \alpha_j(b)$, and to $\min\left(v_i, \rho_j^s(\alpha_j(b), \max(\alpha_j(b), \alpha_i(b)))\right)$ if $v_i > \alpha_j(b)$. (iii) follows. ||

[21] This is the variant (which can be proved as Theorem 2 in Milgrom and Segal 2002 from their Theorem 1) where the requirement that $f(x, .)$ be differentiable for all $x \in X$ is replaced by the requirement that $f(x^*(t), .)$ be differentiable, for any selection $x^*(.) \in X^*(.)$ and almost all $t \in (0, 1)$.

Appendix 2

Lemma A2: *Let i,$j = 1, 2$ be such that $i \neq j$.* If $v - \frac{1-F_j(v)}{f_j(v)}$ *is strictly increasing, then the left-hand partial derivative* $\frac{\partial_l}{\partial v_i}\rho^s(v, v)$ *exists and*

$$\frac{\partial_l}{\partial v_i}\rho^s(v, v) = \frac{1}{2},$$

for all v in $(c, d]$.

Proof: Let v_1, v_2 be such that $c < v_i < v_j \leq d$, with $i \neq j$. Subtracting the definition (1) of $\rho^s(v_1, v_2)$ from v_j and dividing by $v_j - v_i$, we find:

$$1 = \frac{v_j - \rho^s(v_1, v_2)}{v_j - v_i}\left(1 + \frac{1}{f_j(\rho^s(v_1, v_2))}\frac{F_j(v_j) - F_j(\rho^s(v_1, v_2))}{v_j - \rho^s(v_1, v_2)}\right).$$

From the continuity of f_j at v_j and the continuity of ρ^s, $f_j(\rho^s(v_1, v_2))$ tends towards $f_j(v_j)$, when v_i tends towards v_j from below. Since the derivative of F_j at v_j exists and is equal to $f_j(v_j)$, the limit of the ratio $\frac{F_j(v_j)-F_j(\rho^s(v_1,v_2))}{v_j-\rho^s(v_1,v_2)}$ is equal to $f_j(v_j)$. Consequently, the factor between parentheses in the equation above tends towards 2 and the lemma follows. ||

Proof of Lemma 3: Step 1 is a simple definition. To show that Step 2 can be carried out, assume that there exists v such that $\varphi(v) > v$ and consider b_2 in $(\beta_2(\varphi^-(v)), \beta_2(\varphi^+(v)))$ (see Figure 1), where $\varphi^+(v)$ is, as defined in Corollary A1 (Appendix 3), the smallest point of coincidence between β_1 and β_2 to the right of v, that is, $\min\{w \in [v, d] \,|\varphi(w) = w\}$. From Step 1 and Corollary 1 (Section 5), the support of $F_2(.|b_2)$ must be $I = [\rho^s(\alpha_1(b_2), \alpha_2(b_2)), \varphi^+(v)]$. In order for bidder 1 with consumption value $v_1 = \alpha_1\left(\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right)$, with w in I, to propose the same resale price w as in $\mathcal{E}$, w must maximize $(w - v_1)(1 - F_2(w|b_2))$ and hence satisfy (assuming differentiability) the FOC below:

$$\frac{d}{dw}\ln\left(1-F_2\left(w|b_2\right)\right)=\frac{1}{\alpha_1\left(\rho^s\left(\alpha_1\left(.\right),\alpha_2\left(.\right)\right)^{-1}\left(w\right)\right)-w}. \quad \text{(A2.1)}$$

If this necessary FOC is satisfied for all w in the interior of I, it will also be sufficient (since $w-\frac{1-F_2(w|b_2)}{f_2(w|b_2)}$ will then be equal to the increasing function $\alpha_1\left(\rho^s\left(\alpha_1\left(.\right),\alpha_2\left(.\right)\right)^{-1}\left(w\right)\right)$). Integrating this equation (A2.1) in w from the lower extremity of I to v_2 in the interior of I, one finds:

$$F_2\left(v_2|b_2\right)=1-\exp\int_{\rho^s(\alpha_1(b_2),\alpha_2(b_2))}^{v_2}\frac{1}{\alpha_1\left(\rho^s\left(\alpha_1\left(.\right),\alpha_2\left(.\right)\right)^{-1}\left(w\right)\right)-w}dw. \quad \text{(A2.2)}$$

That equation (A2.2) defines a probability distribution over I will be proved if it is proved that the limit of its right-hand side tends towards one as v_2 tends towards $\varphi^+\left(v\right)$. Since it is well known that the integral $\int_{\rho^s(\alpha_1(b_2),\alpha_2(b_2))}^{\varphi^+(v)}\frac{1}{\varphi^+(v)-w}dw$ diverges and since $\frac{w-\alpha_1\left(\rho^s(\alpha_1(.),\alpha_2(.))^{-1}(w)\right)}{\varphi^+(v)-w}=-1+\frac{\varphi^+(v)-\alpha_1\left(\rho^s(\alpha_1(.),\alpha_2(.))^{-1}(w)\right)}{\varphi^+(v)-w}$, it suffices to prove that $\frac{\varphi^+(v)-w}{\varphi^+(v)-\alpha_1\left(\rho^s(\alpha_1(.),\alpha_2(.))^{-1}(w)\right)}$ is bounded away from zero as w tends towards $\varphi^+\left(v\right)$ from below; or, equivalently, through the change of variables $v_1=\alpha_1\left(\rho^s\left(\alpha_1\left(.\right),\alpha_2\left(.\right)\right)^{-1}\left(w\right)\right)$, that $\frac{\varphi^+(v)-\rho^s(v_1,\varphi(v_1))}{\varphi^+(v)-v_1}$ is bounded away from zero as v_1 tends towards $\varphi^+\left(v\right)$ from below.

However, this last ratio is equal to the sum $\frac{\varphi^+(v)-\rho^s\left(v_1,\varphi^+(v)\right)}{\varphi^+(v)-v_1}+\frac{\rho^s\left(v_1,\varphi^+(v)\right)-\rho^s(v_1,\varphi(v_1))}{\varphi^+(v)-v_1}$. The second term is nonnegative and, from $\frac{\partial_l}{\partial v_2}\rho^s\left(\varphi^+\left(v\right),\varphi^+\left(v\right)\right)=\frac{1}{2}$ in Lemma A2 above, the first term tends towards $\frac{1}{2}$. Consequently, it is bounded away from zero and (A2.2) defines a probability distribution[22].

Step 3 leads to a bidding strategy of bidder i if and only if the marginal distribution F_i^* of the joint distribution generated by $F_i\left(.|.\right)$ and $F_i\alpha_i$ is equal to the actual distribution F_i of bidder i's consumption value. To show that this is indeed the case, assume, as in the previous paragraph, that $x=s$

[22] The proof for the other bargaining procedure is similar and makes use of $\frac{\partial_r}{\partial v_2}\rho^b\left(\varphi^-\left(v\right),\varphi^-\left(v\right)\right)=\frac{1}{2}$.

and there exists v such that $\varphi(v) > v$ (see Figure 1). From Step 2, (A2.1) or, equivalently, (A2.3) below holds true, for all w in $(\varphi^-(v), \varphi^+(v))$ and all b_2 in $\left[\beta_2(\varphi^-(v)), \rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right]$ ($\varphi^-(v)$ is defined in Corollary 1, Section 5):

$$\left(w - \alpha_1\left(\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right)\right) f_2(w|b_2) = 1 - F_2(w|b_2), \quad \text{(A2.3)}$$

where $f_2(.|b_2)$ denotes the derivative of $F_2(.|b_2)$. Integrating (A2.3) in b_2 according to $F_2\alpha_2$ over $\left[\beta_2(\varphi^-(v)), \rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right]$, one finds (A2.4) below (using $F_2^*(\varphi^-(v)) = F_2(\varphi^-(v))$, which follows from Steps 1 and 2):

$$\left(w - \alpha_1\left(\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right)\right) f_2^*(w) = F_2\left(\alpha_2\left(\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right)\right) - F_2^*(w). \quad \text{(A2.4)}$$

However, the obvious equality $\rho^s(\alpha_1(.), \alpha_2(.))\left(\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right) = w$ implies:

$$\left(w - \alpha_1\left(\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right)\right) f_2(w) = F_2\left(\alpha_2\left(\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right)\right) - F_2(w). \quad \text{(A2.5)}$$

Subtracting (A2.5) from (A2.4) gives:

$$\frac{d}{dw}\left(F_2^*(w) - F_2(w)\right) = \frac{F_2(w) - F_2^*(w)}{w - \alpha_1\left(\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(w)\right)}.$$

From this differential equation, the difference $F_2^* - F_2$ either vanishes or is of constant sign over the interval $(\varphi^-(v), \varphi^+(v))$. From this same differential equation, a constant sign implies strict monotonicity, which contradicts the equality, from Steps 1 and 2, between F_2^* and F_2 at both extremities of this interval. Consequently, F_2^* and F_2 coincide everywhere and $G_2(.|.)$ is a bidding strategy of bidder 2[23].

[23] For all v_2 in $(\varphi^-(v), \varphi^+(v))$, taking the derivative with respect to v_2 of (A2.2), integrating with respect to b_2, distributed according to $F_2\alpha_2$, from b in $\left(\beta_2(\varphi^-(v_2)), \rho^s(\alpha_1(.), \alpha_2(.))^{-1}(v_2)\right)$ to $\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(v_2)$, and dividing by $f_2(v_2)$

I now show that $\delta_i(v_i; b_j)$ in Step 4 gives optimal resale prices. When mutually profitable resale is not possible, any resale price at least equal to the consumption value v_i, as $\delta_i(v_i; b_j)$ always is, is optimal. I may thus focus on the case where profitable resale is possible. Assume first $b_j \leq \beta_i(v_i)$. From Step 3, $\rho_i^s(v_i, \alpha_j \beta_i(v_i))$, the value of $\delta_i(v_i; b_j)$, is optimal for bidder i when mutually profitable resale is possible. Assume next $b_j > \beta_i(v_i)$ or, equivalently, $\alpha_i(b_j) > v_i$. If $\alpha_i(b_j) \leq \alpha_j(b_j)$, $\delta_i(v_i; b_j)$ is equal to $\rho_i^s(\alpha_i(b_j), \alpha_j(b_j))$. Since it is the minimum of the support of $F_j(.|b_j)$ and, since it is optimal for bidder i with the higher consumption value $\alpha_i(b_j)$, it is also optimal for the consumption value v_i. If $\alpha_i(b_j) > \alpha_j(b_j)$, $F_j(.|b_j)$ is concentrated at $\alpha_j(b_j)$, which is then optimal for bidder i if mutually profitable resale is possible, that is, if $v_i < \alpha_j(b_j)$. Since $\delta_i(v_i; b_j)$ is equal to $\alpha_j(b_j)$ in this case, the proof is complete. ||

Proof of Lemma 5: From Lemma 4 and Step 4 of the construction of $\mathcal{E}'$ in Lemma 3, the resale price and, thus, the net value of the auction loser do not depend on his bid. (i.1) follows, which immediately implies (i.2).

(ii) holds true because resale occurs with probability one after a tie at b such that $\alpha_1(b) \neq \alpha_2(b)$ is broken inefficiently. In fact, assume, for example, $\alpha_2(b) > \alpha_1(b)$. Then, according to the revised beliefs, bidder 1's consumption value is $\alpha_1(b)$ and the minimum of the support of bidder 2's consumption value is $\rho^s(\alpha_1(b), \alpha_2(b))$. With probability one the resale offer $\rho^s(\alpha_1(b), \alpha_2(b))$ from bidder 1, if he wins the tie, is accepted by bidder 2 and is thus equal to the bidders' net values.

Bidder i obtains the conditional net expected payoff $\int u_i^s(v_i, v_j; b_j, b_j)\, dF_j(v_j|b_j)$ if he observes b_j and proposes his optimal resale price after winning the auc-

give an explicit expression for $1 - G_2(b|v_2)$, from which one finds:

$$G_2(b|v_2) = 1 - \frac{\int_b^{\rho^s(\alpha_1(.),\alpha_2(.))^{-1}(v_2)} \exp \int_{\rho^s(\alpha_1(b_2),\alpha_2(b_2))}^{v_2} \frac{dw}{\alpha_1\left(\rho^s(\alpha_1(.),\alpha_2(.))^{-1}(w)\right) - w} dF_2 \alpha_2(b_2)}{f_2(v_2)\left(v_2 - \alpha_1\left(\rho^s(\alpha_1(.), \alpha_2(.))^{-1}(v_2)\right)\right)}.$$

tion. It is thus the maximum of the net expected payoff he obtains when he proposes p_i, over all possible resale prices p_i. Since, for any fixed p_i, his net expected payoff is nondecreasing in his consumption value v_i, so is his optimal net expected payoff and (iii) follows. $||$

Proof of Corollary 3: Bidder i's expected net payoffs in the FPA and the SPA can be written as follows:

FPA:

$$\int_d^{b_i} \int u_i^s (v_i, v_j; b_j, b_j) \, dF_j (v_j | b_j) \, dF_j \alpha_j (b_j) - \int_c^{b_i} b_i dF_j \alpha_j (b_j) ;$$

SPA:

$$\int_c^{b_i} \left(\int u_i^s (v_i, v_j; b_j, b_j) \, dF_j (v_j | b_j) - b_j \right) dF_j \alpha_j (b_j) - \int_c^{d} \int u_i^s (v_i, v_j; b_j, b_j) \, dF_j (v_j | b_j) \, dF_j \alpha_j (b_j) .$$

Proceeding then as in the proof in Section 4 of Theorems 1 and 2, one obtains that the equilibrium bids in $\mathcal{E}$ of bidder i are also optimal when bidder j follows his strategy in $\mathcal{E}'$ and disclosure is full. Proceeding as in the proof of Corollary 1 (Section 5), one also obtains the same optimal bids for bidder i. Since the sets of these bids are the supports of the bidding strategies $G_i (.|.)$, $i = 1, 2$, $\mathcal{E}'$ in Corollary 3 is a PBE under full disclosure.

Moreover, from the expression above for the expected payoff in the SPA, Lemma 5 (i.2), (ii), and (iii), and the equality $\rho^s (\alpha_1 (b), \alpha_2 (b)) = b$, any of a bidder's equilibrium bids in the SPA wins against bids that contribute nonnegatively to his net expected payoff and loses against those that contribute nonpositively. A bidder always observes the bid from his opponent before proposing a resale price. Consequently, even if he was allowed to, a bidder would have no incentive to change his bid after learning his opponent's bid, and I have proved Corollary 3 (iii).

The final allocation is the same in $\mathcal{E}'$ as in $\mathcal{E}$. Assume, for example, that bidder 1's consumption value v_1 is such that $\varphi (v_1) \geq v_1$, where $\varphi = \alpha_2 \beta_1$.

Then, bidder 1 bids $\beta_1(v_1)$. If λ_φ^s is as defined in Corollary 2 (Section 5), $v_1 \leq \lambda_\varphi^s(v_1) = \rho^s(v_1, \varphi(v_1)) \leq \varphi(v_1)$. If $v_2 \leq \lambda_\varphi^s(v_1)$, Step 2 of the construction in Lemma 3 implies that bidder 2 with consumption value v_2 bids at most $\max\left(\beta_1(\varphi^-(v_1)), \rho^s(\alpha_1(.), \alpha_2(.))^{-1}(v_2)\right)$, which is not larger than $\beta_1(v_1)$. Consequently, neglecting ties, bidder 2 loses the auction and refuses bidder 1's offer. If $v_2 > \lambda_\varphi^s(v_1)$, bidder 2 accepts bidder 1's resale offer when bidder 1 wins and there is no profitable resale when bidder 2 wins.

Moreover, the bidders with the lowest possible consumption value c obtain the same expected payoff–zero–from both equilibria.

In my model, no private information is ever exogenously released: the information link between the auction stage and the resale stage, through the bidders' bids, is endogeneous and part of an equilibrium. Moreover, the bidders and the auctioneer do not discount future revenues nor payments, that is, they use the common discount factor $\delta = 1$. I may thus apply Myerson (1981, Lemma 2, p. 63)'s result according to which a bidder's expected payoff conditional on his consumption value depends only on the equilibrium final allocation of the item and his expected payoff for the smallest possible consumption value. Consequently, these interim expected payoffs are the same in $\mathcal{E}'$ as in $\mathcal{E}$. Because, from Step 3 of the construction in Lemma 3, the marginal bid distributions are the same, I have proved Corollary 3 (i).

Since, from Step 1 in Lemma 3, the more aggressive, conditional on his consumption value, bidder submits the same bid and faces the same bid distribution in both equilibria, his expected payoff from the auction stage and his probability of winning are also the same. From Step 2 in Lemma 3, this more aggressive bidder makes the same resale offer, which is larger than his consumption value. In order to generate the same total expected payoffs, the probabilities of resale must be the same in both equilibria and I have proved Corollary 3 (ii).

Because the offer from the auction winner does not depend, along the equilibrium path, on the bid from the auction loser, it is in the interest of no bidder to deviate from the bidding strategies in $\mathcal{E}'$ if the bids are not fully disclosed. In fact, a deviation by a bidder after which he loses the auction has the same result as under full disclosure. A deviation after which he wins the auction is at most as profitable, since less information is available to make an optimal proposal at resale. Corollary 3 (iv) follows. ||

Appendix 3

Theorem A1: *Let $v+\frac{F_1(v)}{f_1(v)}$ and $v+\frac{F_2(v)}{f_2(v)}$ be strictly increasing over $[c,d]$. Let φ be a strictly increasing continuous function over $[c,d]$ such that $\varphi(c)=c$ and $\varphi(d)=d$. Let $\beta_1,\ \beta_2,\gamma_1,\gamma_2$ be the following bidding and resale-offer functions:*

$$\begin{aligned}\beta_1(v) &= \rho^b(v,\varphi(v)),\\ \beta_2(v) &= \rho^b\left(\varphi^{-1}(v),v\right),\end{aligned}$$

for all v in $[c,d]$;

$$\gamma_i(v;b)=\rho_i^b(v,\min(v,\alpha_j(b))),$$

where α_j is the extended inverse of β_j, $i,j=1,2$, $i\neq j$, for all (v,b) in $[c,d]\times[c,+\infty)$. Then, $(\beta_1,\gamma_1;\beta_2,\gamma_2)$ is a PBE of the SPA under the no-disclosure regime when the auction loser is the resale-price maker. Moreover, the following equalities hold true:

$$\begin{aligned}\alpha_2\beta_1 &= \varphi,\\ \rho^b(\alpha_1(b),\alpha_2(b)) &= b,\end{aligned}$$

for all v in $[c,d]$ and b in $[\beta_1(c),\beta_1(d)]=[\beta_2(c),\beta_2(d)]=[c,d]$.

Theorem A2: *Let $v+\frac{F_1(v)}{f_1(v)}$ and $v+\frac{F_2(v)}{f_2(v)}$ be strictly increasing over*

$[c,d]$. *Let* β_1, $\beta_2, \gamma_1, \gamma_2$ *be the following bidding and resale-offer functions:*

$$\beta_i(v) = \frac{\int_0^{F_i(v)} \rho^b \left(F_1^{-1}(q), F_2^{-1}(q)\right) dq}{F_i(v)}, \text{ (A3.1)}$$

for all v *in* $[c,d]$*;*

$$\gamma_i(v;b) = \rho_i^b(v, \min(v, \alpha_j(b))),$$

where α_j *is the extended inverse of* β_j*,* $i,j = 1,2$*,* $i \neq j$*, for all* (v,b) *in* $[c,d] \times [c,+\infty)$*. Then,* $(\beta_1, \gamma_1; \beta_2, \gamma_2)$ *is a PBE of the FPA under the no-disclosure regime when the auction loser is the resale-price maker. The following equality holds true:*

$$\alpha_2 \beta_1 = F_2^{-1} F_1. \text{ (A3.2)}$$

Moreover, β_1, β_2 *are the unique bidding functions part of a pure PBE that are, over* $(c,d]$*, strictly increasing, differentiable, and such that* $\beta_1(c) = \beta_2(c) = c$*.*

Corollary A1: *Let* $(\beta_1, \gamma_1; \beta_2, \gamma_2)$ *be a PBE as in Theorem A1 or Theorem A2 and let* φ *be equal to* $\alpha_2\beta_1$*. Let* φ^+ *be the function defined as follows:*

$$\varphi^+(v) = \min\{w \in [v,d] \,|\, \varphi(w) = w\},$$

for all v *in* $[c,d]$*. Then all bids in*

$$\left[\rho^b(\alpha_1(.), \alpha_2(.))^{-1}(v), \beta_1(\varphi^+(v))\right]$$

are optimal for bidder 1 (2) with consumption value v *in* $[c,d]$ *such that* $\varphi(v) > (<) v$*.*

Corollary A2: *Let* $(\beta_1, \gamma_1; \beta_2, \gamma_2)$ *be a PBE as in Theorem A1 or Theorem A2 and let* φ *be equal to* $\alpha_2\beta_1$*. Let* λ_φ^b *be the function defined over* $[c,d]$ *as in Section 7. If bidder 1's and bidder 2's consumption values*

v_1, v_2 *in* $[c, d]$ *are such that* $v_2 <(>) \lambda_\varphi^b (v_1)$*, then the equilibrium eventually allocates the item to bidder 1 (2).*

Corollary A3: *Let* $\mathcal{E}$ *be a PBE of an auction (FPA or SPA) under the no-disclosure regime as in Theorem A1 or A2. Then, there exists a PBE* $\mathcal{E}'$ *of the auction under the full-disclosure regime such that:*

(i) The bid marginal distributions, the interim total expected payoffs, and the final allocation are the same as in $\mathcal{E}$*;*

(ii) Conditionally on the consumption value of the auction loser, strictly profitable resale takes place with the same probability as in $\mathcal{E}$ *and, when this probability is different from zero, at the same price;*

(iii) If the auction is the SPA, the auction outcomes–the bids and the allocation before resale–are posterior implemented by $\mathcal{E}'$ *or, for short,* $\mathcal{E}'$ *is posterior implementable.*

(iv) There exists a PBE of the auction under no disclosure[24] *with the same bidding strategies and, along the equilibrium path, the same offers from the resale-price maker as* $\mathcal{E}'$*.*

Appendix 4

A Class of Examples Where the Equilibrium Bid Distributions Do not Increase When the Bargaining Power at Resale Goes From the Auction Loser to the Auction Winner:

Consider the example where there exists q in $(0, 1)$ such that $F_1^{-1}(p) < F_2^{-1}(p)$, for all p in $(0, q]$, F_1 is strictly convex over $\left[0, F_1^{-1}(q)\right]$, and F_2 is strictly concave over $\left[0, F_2^{-1}(q)\right]$ (such an example obviously exists). Then, the definitions (Definitions 1 and 2) of the seller's and buyer's optimal resale price functions ρ^s and ρ^b readily imply $\rho^b\left(F_1^{-1}(p), F_2^{-1}(p)\right) > \frac{F_1^{-1}(p)+F_2^{-1}(p)}{2} >$

[24](iv) also applies to any other disclosure policy.

$\rho^s\left(F_1^{-1}(p), F_2^{-1}(p)\right)$, for all p in $(0,q]$. If I denote by β_i^s and β_i^b the equilibrium bidding functions of the FPA under no disclosure when the resale price is chosen by the auction winner (the seller) and the auction loser (the buyer), I find, from the formulas in Theorems 2 (Section 3) and A2 (Appendix 3):

$$\begin{aligned}\beta_i^s(v) &= \frac{\int_0^{F_i(v)} \rho^s\left(F_1^{-1}(q), F_2^{-1}(q)\right) dq}{F_i(v)} \\ &< \frac{\int_0^{F_i(v)} \rho^b\left(F_1^{-1}(q), F_2^{-1}(q)\right) dq}{F_i(v)} = \beta_i^b(v),\end{aligned}$$

for all v_i in $\left(0, F_i^{-1}(q)\right)$. Consequently, for small bids, both bidders' bid distributions shift *downwards* if the auction winner becomes the resale-price maker. From Corollaries 3 and A3, the same conclusion applies to the full-disclosure regime.

Appendix 5

Proof of Lemma 6: The first statement, pertaining to the case $\varphi(v) = v$, is immediate. To prove the second statement, assume, for example, $\varphi(v) > v$. The definitions of λ_φ^s and λ_φ^b and the equality $\varphi = F_2^{-1}F_1$ imply $\lambda_\varphi^s(v) = \rho^s\left(v, F_2^{-1}F_1(v)\right)$ and $v = \rho^b\left(F_1^{-1}F_2\left(\lambda_\varphi^b(v)\right), \lambda_\varphi^b(v)\right)$, that is:

$$v = \lambda_\varphi^s(v) - \frac{F_1(v) - F_2\left(\lambda_\varphi^s(v)\right)}{f_2\left(\lambda_\varphi^s(v)\right)}, \quad \text{(A5.1)}$$

$$v = \lambda_\varphi^b(v) + \frac{F_2\left(\lambda_\varphi^b(v)\right) - F_1(v)}{f_1(v)}. \quad \text{(A5.2)}$$

From the definitions of ω_1^b, ω_2^b, (A5.1) is equivalent to (A5.3) below:

$$\omega_2^b\left(\lambda_\varphi^s(v)\right) = \omega_1^b(v) + (1 - F_1(v))\left(\frac{1}{f_1(v)} - \frac{1}{f_2\left(\lambda_\varphi^s(v)\right)}\right). \quad \text{(A5.3)}$$

If $\lambda_\varphi^s(v) < \psi(v)$, (12) and (A5.3) imply $f_1(v) > f_2\left(\lambda_\varphi^s(v)\right)$. Since $F_1(v) - F_2\left(\lambda_\varphi^s(v)\right) > 0$, (A5.1) then implies:

$$v < \lambda_\varphi^s(v) - \frac{F_1(v) - F_2\left(\lambda_\varphi^s(v)\right)}{f_1(v)}. \quad \text{(A5.4)}$$

Combining (A5.4) with (A5.2) and rearranging give:

$$\lambda_\varphi^b(v) + \frac{F_2\left(\lambda_\varphi^b(v)\right)}{f_1(v)} < \lambda_\varphi^s(v) + \frac{F_2\left(\lambda_\varphi^s(v)\right)}{f_1(v)},$$

which immediately implies $\lambda_\varphi^b(v) < \lambda_\varphi^s(v)$. The proof is complete once it is noticed that all the implications above are actually equivalences. ||

Proof of Lemma 7: Assume that there exists v such that $\varphi(v) > v$. Suppose λ_φ^s and ψ or, equivalently, from Lemma 6, λ_φ^b and ψ are identical over $(\varphi^-(v), \varphi^+(v))$, where $\varphi^-(v)$ and $\varphi^+(v)$ are as in Corollaries 1 (Section 5) and A1 (Appendix 3). From (A5.3) and (A5.1), $f_1(w) = f_2(\psi(w))$ and $F_1(w) - F_2(\psi(w)) = f_1(w)(\psi(w) - w)$, for all w in $(\varphi^-(v), \varphi^+(v))$. Because its derivative then vanishes, $(F_1(w) - F_2(\psi(w)))(\psi(w) - w)$ is constant over this interval. However, this is impossible since it tends towards zero at the extremities, while being strictly positive in the interior (since $w < \lambda_\varphi^s(w) = \psi(w) < \varphi(w) = F_2^{-1}F_1(w)$). ||

Appendix 6

In any of the PBE's in this paper and for any consumption value v, either both bidders submit the same bid with certainty at v, or one bidder is more aggressive than the other at v, that is, his bid is larger than the other's with probability one. This is obvious for the pure PBE's. It also holds true in the behavioral PBE's because the bid supports at any consumption value are convex and either identical or disjoint. Consequently, when the bidders' strategies differ at v, one bidder is an equilibrium resale-price maker–the more (less) aggressive bidder when the auction winner (loser) sets the resale price–and the other bidder is an equilibrium resale-price taker.

The PBE's of the SPA are somewhat independent of the consumption value distribution of the equilibrium resale-price maker. When, for example, $\varphi(v) > v$ and the auction winner chooses the resale price, the bidding strategies remain part of a PBE if the probability distribution of bidder 1's consumption value is changed in a neighborhood of v. In fact, in Theorem 1 (Section 3), the bids and the resale prices the bidders submit along the equilibrium path depend only on the values of the optimal resale price function ρ^s at (v_1, v_2) with $\varphi(v_1) = v_2$. However, $\rho^s(v_1, v_2)$, with $v_2 > v_1$, is bidder 1's optimal resale price and hence depends only on the distribution of bidder 2's consumption value. The construction (Lemma 3) of the equivalent PBE under full disclosure in Corollary 3 only uses the probability distribution of the equilibrium resale-price taker in order to transform his bidding function into a behavioral bidding strategy. Moreover, when $\varphi(v) = v$, whatever the consumption value distributions are, both bidding strategies are pure and equal (to φ) at v. Corollary A4 below follows.

Corollary A4:

(i) Let $v - \frac{1-F_i(v)}{f_i(v)}$ *and* $v - \frac{1-F_i'(v)}{f_i'(v)}$ *be strictly increasing, for* $i = 1, 2$. *In the SPA where the auction winner is the resale-price maker and where the bidders' consumption values are distributed according to* F_1, F_2, *let* $\mathcal{E}$ *be a PBE as in Theorem 1 (under no disclosure) or as constructed in the proof of Corollary 3 (under full disclosure). Then the bidding strategies in* $\mathcal{E}$ *remain part of a PBE when the bidders' consumption values are distributed according to* F_1' *and* F_2' *if* F_1' *is equal to* F_1 *over* $\{v_1 \in [c,d] \,|\, \varphi(v_1) < v_1\}$ *and* F_2' *is equal to* F_2 *over* $\{v_1 \in [c,d] \,|\, \varphi(v_1) > v_1\}$.

(ii) Let $v + \frac{F_i(v)}{f_i(v)}$ *and* $v + \frac{F_i'(v)}{f_i'(v)}$ *be strictly increasing, for* $i = 1, 2$. *In the SPA where the auction loser is the resale-price maker and where the bidders' consumption values are distributed according to* F_1, F_2, *let* $\mathcal{E}$ *be a*

PBE as in Theorem A1 (under no disclosure) or as constructed in the proof of Corollary A3 (under full disclosure). Then the bidding strategies in $\mathcal{E}$ remain part of a PBE when the bidders' consumption values are distributed according to F_1' and F_2' if F_1' is equal to F_1 over $\{v_1 \in [c,d] \,|\varphi(v_1) > v_1\}$ and F_2' is equal to F_2 over $\{v_1 \in [c,d] \,|\varphi(v_1) < v_1\}$.

As it can be easily checked, the assumption that the function φ be strictly increasing and continuous is not necessary for the results about the SPA (Theorems 1, A1, Corollaries 3, and A3). In fact, the proofs and constructions go through when φ is only assumed to be nondecreasing. Corollary A5 below holds true when an inverse φ^{-1} of φ is a function such that v lies between the limits[25] of $\varphi(w)$ for w tending from below and from above to $\varphi^{-1}(v)$, for all v in $[c,d]$. Although φ may be constant or discontinuous in Corollary A5, β_1 and β_2 are strictly increasing and their inverses α_1 and α_2 are uniquely defined and continuous. Minor adjustments of some definitions in the previous proofs are necessary to carry over to such a more general φ. For example, Corollary 1 now applies to bidder 1 with consumption value v such that[26] $\lim_{u \to <v} \varphi(u) < v$, in which case the lower extremity $\varphi^-(v)$ of the set of optimal bids $[\varphi^-(v), v]$, over which bidder 1 randomizes under full disclosure, is the largest fixed point w smaller than v of the correspondence $\Phi(w) = [\lim_{u \to <w} \varphi(u), \lim_{u \to >w} \varphi(u)]$.

Corollary A5[27]**:** *The results Theorems 1, A1, and Corollaries 3, A3 about the SPA hold true if the function φ from $[c,d]$ to $[c,d]$ is only required to be nondecreasing and if φ^{-1}, in the formulas of Theorems 1 and A1, denotes*

[25] With the conventions $\lim_{w \underset{<}{\to} c} \varphi(w) = c$ and $\lim_{w \underset{>}{\to} d} \varphi(w) = d$.

[26] It applies to bidder 2 with consumption value v such that $\lim_{u \to <v} \varphi^{-1}(u) < v$.

[27] The supports of the bid distributions are not convex (intervals) when φ is constant or discontinuous. Under full disclosure, a bidder's revised beliefs are not uniquely determined when he observes a bid in one of the gaps of his opponent's equilibrium bid distribution. Taking for these beliefs off the equilibrium path the particular distributions from Section 6 and Appendix 2, which depend on the (uniquely defined) inverses α_1 and α_2, ensures the equilibrium is perfect Bayesian.

an inverse of φ*.*

As an example of application of Corollary A5, consider the function φ such that $\varphi(v_1) = \theta^*$, for all v_1 in $[c, \theta^*]$, and $\varphi(v_1) = v_1$, for all v_1 in $[\theta^*, d]$. Then, the equilibrium bidding functions under no disclosure when the auction winner is the resale-price maker are β_1 and β_2 such that $\beta_1(v_1) = \rho^s(v_1, \theta^*)$, for all v_1 in $[c, \theta^*]$, and $\beta_1(v_1) = v_1$, for all v_1 in $[\theta^*, 1]$; $\beta_2(v_2) = \rho^s(c, v_2)$, for all v_2 in $[c, \theta^*)$, $\beta_2(\theta^*)$ belongs to[28] $[\rho^s(c, v_2), \theta^*]$, and $\beta_2(v_2) = v_2$, for all v_1 in $(\theta^*, 1]$. In the equivalent PBE under full disclosure, constructed as in Lemma 3 (Section 6), the bidders' bidding strategies are the same as under no disclosure except that bidder 2 randomizes over $[c, v_2]$ when his consumption value belongs to $[c, \theta^*]$. In this PBE, as in the initial equilibrium under no disclosure, bidder 1 with consumption value v_1 offers after winning the resale price $\rho^s(v_1, \theta^*)$ if v_1 is in $[c, \theta^*]$ and v_1 otherwise, no matter what bidder 2's bid is. From Corollary A4 also, the bidding strategies of this PBE under full disclosure remain part of a PBE if F_1 is changed to any distribution F_1' (with increasing virtual value)[29].

Another PBE under full disclosure can be obtained by changing differently bidder 2's bidding strategy over $[c, \theta^*]$. Assume that bidder 2 now submits c when his consumption value belongs to $[c, \theta^*]$. Since c was among his optimal bids, bidder 2 has no incentive to deviate. If bidder 1's revised beliefs after winning against a bid b_2 in $[0, \theta^*)$ is the restriction of F_2 over $[c, \theta^*]$, the same resale prices remain optimal for him. Bidder 1 has no incentive to deviate from his bidding strategy because by submitting a bid in $(c, \rho^s(c, \theta^*))$, where bidder 2's bid distribution has changed, he now obtains the same expected payoff he previously obtained by submitting a bid in $(\rho^s(c, \theta^*), \theta^*)$[30]. This

[28] $\beta_2(\theta^*) = \rho^s\left(\varphi^{-1}(\theta^*), \theta^*\right)$, for an inverse φ^{-1} of φ. Since $\varphi^{-1}(\theta^*) \in [c, \theta^*]$, $\beta_2(\theta^*)$ belongs to $[\rho^s(c, v_2), \theta^*]$.

[29] If F_1' is concentrated at c, they still remain part of a PBE and the alternative construction below gives the equilibrium with no discounting in Garratt and Tröger (2006).

[30] This is also true of deviating to c if the tie is broken in favor of bidder 1. For any

new PBE is the equilibrium Garratt, Tröger, and Zheng (2006) obtain in the case without discounting[31][32].

Appendix 7

Proof of Corollary 8 (ii): The first part of (ii) is proved in the main text. I now prove the second part. In the $(\varsigma k_1, \varsigma k_2)$-PA, $\beta_1(v)$, for example, is, from (15), equal to:

$$\beta_1(v) = \frac{\int_c^v \rho^x \left(w, F_2^{-1}\left(F_1(w)^{k_1/k_2}\right)\right) dF_1(w)^{1/\varsigma k_2}}{F_1(v)^{1/\varsigma k_2}},$$

for all v in $(c, d]$. Since the probability distribution $\left(\frac{F_1}{F_1(v)}\right)^{1/\varsigma k_2}$ over $[c, v]$ dominates $\left(\frac{F_1}{F_1(v)}\right)^{1/\varsigma' k_2}$, for all $0 < \varsigma < \varsigma'$, $\beta_1(v)$ increases as ς decreases. When ς tends towards zero, the distribution $\left(\frac{F_1}{F_1(v)}\right)^{1/\varsigma k_2}$ converges weakly towards the degenerate distribution concentrated at v and, consequently, $\beta_1(v)$ tends towards $\rho^x(v, \varphi(v))$, where $\varphi = F_2^{-1} F_1^{k_1/k_2}$. Similarly, $\beta_2(v)$ increases and converges towards $\rho^x\left(\varphi^{-1}(v), v\right)$ when ς tends towards zero. ||

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other tie breaking rule, bidder 1 still has no incentive to deviate to c since his expected payoff would be strictly smaller .

[31] A similar construction when the auction loser sets the resale price gives the bidding functions $\beta_1(v_1) = v_1$, for all v_1 in $[c, \theta^*)$; $\beta_1(v_1) = d$, for all v_1 in $(\theta^*, d]$; $\beta_2(v_2) = v_2$, for all v_2 in $[c, \theta^*)$; $\beta_2(v_2) = \rho^b(\theta^*, v_2)$, for all v_2 in $(\theta^*, d]$.

[32] The bidding functions β_1 and β_2 where β_1 takes the constant value d and β_2 takes the constant value c can always be completed into PBE's of the SPA under no disclosure and full disclosure. In fact, bidder 1 wins the auction with probability one at the price equal to the lowest consumption value c and can make additional profits from resale, while if he lost the auction, by submitting c and losing the tie, he could only obtain the item at resale at a price at least equal to c. If bidder 2 submitted a bid equal to or higher than d, he could not strictly recoup his auction losses from his resale profits. Since the auctioneer obtains revenues equal to c from such an equilibrium, the lower extremities of the auctioneer's equilibrium revenues are the same under both regimes. From Corollaries 5 and 6, the upper extremities are also equal (under Assumptions A^s and A^b).

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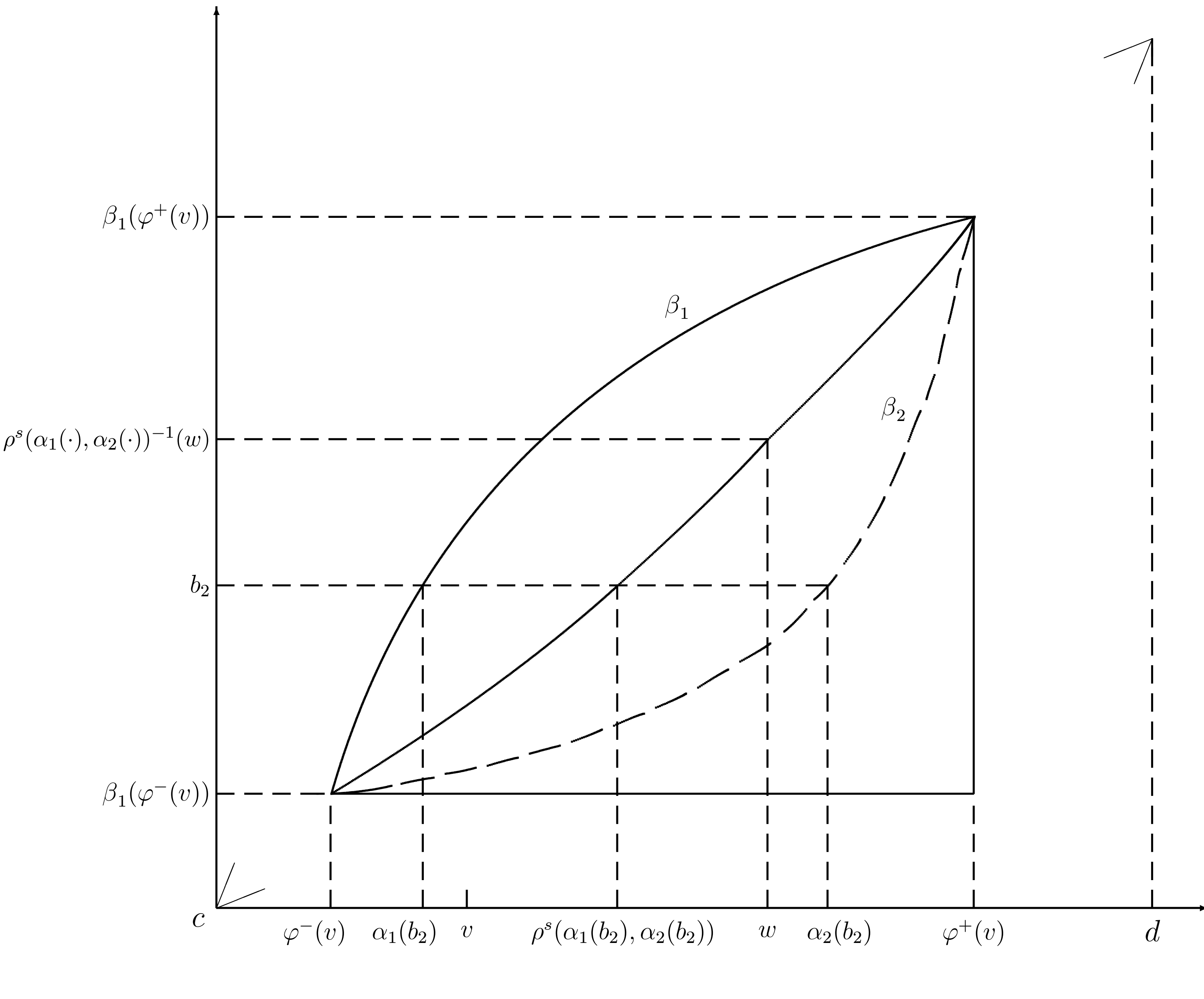
Figure 1



$\beta_1(\varphi^+(v))$
β_1
$\rho^s(\alpha_1(\cdot),\alpha_2(\cdot))^{-1}(w)$
β_2
b_2
$\beta_1(\varphi^-(v))$
c
$\varphi^-(v)$
$\alpha_1(b_2)$
v
$\rho^s(\alpha_1(b_2),\alpha_2(b_2))$
w
$\alpha_2(b_2)$
$\varphi^+(v)$
d